

**A MATHEMATICAL MODEL
FOR
LEAST SQUARES
POINT DETERMINATION
FROM
CADASTRAL DATA**

by

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Thesis presented in partial fulfilment of the requirements for the degree of

Master of Science in Engineering (M.Sc.(Eng))

Department of Surveying and Geodetic Engineering

University of Cape Town

February 1997

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ACKNOWLEDGEMENTS

The author records his sincere thanks and appreciation to the following:

Mr M.B. Barry of the Department of Surveying and Geodetic Engineering at the University of Cape Town for his guidance in the preparation of this thesis. Professor H. Rüther of the Department of Surveying and Geodetic Engineering at the University of Cape Town for his help and advice with least squares.

Mr W.D. Roth for help and advice with the least squares aspects of this thesis, and for the proof reading of this document. Mr M. Fielding from the Professional Communication Unit at UCT for proof reading this document, and his detailed and very helpful comments.

From Transnet Land Surveying Services Johannesburg. All the staff for support and encouragement. Mr A.A. Pienaar and Mr P.D. Herbst for proof reading parts of the document. Mr M.G. du Plessis for allowing his staff to assist me in spite of deadlines on urgent projects. Miss J. Lombard for the preparation of Appendix A and both she and Mr D. Dyers for assistance with the creation of the data sets. The department for making the data and the programming resources available.

My wife Antoinette, and our children Aaron and Matthew, for their patience and understanding over the last two years, (or more!).

TERMS OF REFERENCE

This thesis is a requirement of the course SUR505Z, in part fulfilment of the requirements of the degree of Master of Science in Engineering.

CERTIFICATE OF ORIGINAL WORK

I certify that this is my own original work, and that the work of others has been acknowledged where appropriate.

Lance Nel

SYNOPSIS

INTRODUCTION

Transnet possesses approximately 60000 land parcels covering the whole of the South African rail network. These parcels, or polygons, will form the base map in a corporate Geographic Information System (GIS) which will be used for property administration and general railroad operations management. Accuracy requirements are high and unique coordinate values have to be determined for each polygon apex before data are input into the GIS. Railroad property portfolios are characteristically configured in long thin strips. The solution therefore has to cope with poor geometry. The method used to determine apex coordinates must be time and cost efficient, and produce acceptable levels of precision.

This study examines the feasibility of using a particular mathematical model with the least squares method in the partial automation of the determination of unique points from sets of differing and, at times, conflicting cadastral data for the creation of a digital cadastral database. The approach is not a mathematically rigorous conventional survey network solution, but a pragmatic application of least squares and network principles, to suit the nature and limitations of the data.

CONCEPTUAL SOLUTION

The concept underlying the solution rests on three foundations.

The first is that data defining the length and direction of each side of every cadastral polygon are either recorded on the land surveyor's diagram, or can be derived from the data (coordinates and/or sides and angles) on such diagrams.

The second is that these data can be combined in a least squares survey network adjustment to calculate unique values for all the polygon apices in a single coordinate system.

The third is that numerous points have been surveyed in the national survey system that can be held fixed in a network adjustment, thereby defining the single survey system and forming the framework into which all other data are adjusted.

AIMS OF THE STUDY

The aims of the study are :-

- The development of a mathematical model, as the basis for a least squares program for determining the coordinates of unique points. This should be done by combining, and resolving inconsistencies in, data from cadastral land surveyors diagrams in diverse formats, units, coordinate systems and precisions.
- To test the methodology on long strips of polygons, to determine if it yields correct coordinates and reliable precision estimators, and to determine if there are circumstances where the methodology may fail in either of these aspects.
- To determine the usefulness of the method, by examining the perceptions of the technicians responsible for the creation of the digital spatial cadastral database in the Johannesburg office of Transnet's Land Surveying Services, with respect to the simplicity and the speed of the process compared with alternative processes. The processes they currently use are assumed representative of alternative processes.

LEAST SQUARES PRINCIPLES

A least squares network solution is based on using all data from all sources in a single calculation process that minimises the corrections to be applied to the observations in order to fit the most probable solution to the unknowns. The problem must be overdetermined, or in other words, there must be redundant observations.

DATA SETS PROCESSED

Two test areas have been used. The first has all of its thirty-six polygons surveyed on the Lo system, which provided an excellent reference, and an opportunity to simulate various data sources through manipulation of the data. The second has most polygons surveyed on the Goldfields system. The manipulated data sets are typical of the linear chains of polygons in Transnet's property portfolio

The data from test area one were first processed with all the trigonometrical beacons in place. This meant that the coordinates were only shifted according to changes in the trigonometrical beacons, and not processed through the least squares network. (Except for two polygons which had no

trigonometrical beacons recorded). The results from this process formed the reference data set with which the results of processing the simulated systems were compared.

Two basic simulations were done. The first involved removing the trigonometrical beacons from a number of polygons, to simulate a local coordinate system, which was then adjusted by the least squares program. The second simulation involved adding a variety of shifts to the same polygons, to simulate disparate data sets, which were then also adjusted by the least squares program.

At this stage it was discovered that certain geometric configurations caused either instabilities or singularities in the solution. Some manipulation of the data resolved the problem. It was, however, realised that there was a flaw in the mathematical model. This model was then changed, which went a long way towards eliminating the problem. The data were reprocessed under the new mathematical model.

DISCUSSION OF RESULTS

All of the results from test area one using the one scale and orientation per polygon program have been summarised into a table titled "Results: Test Area One" and included as Appendices F and G.

It was noticed that insufficient common points between adjacent polygons caused either a singularity (no common points) and the adjustment failed to execute, or caused a deterioration in the precision estimators and instability in the determined coordinates (1 common point).

This problem was solved by adding points to one of the polygons to create extra common points. If the polygons were from the same survey record, or a compatible survey system, then this was a simple task. If they were from different survey records then the task required some more extensive reconstruction calculations. (See Section 9.1.3).

A second problem that was identified was that a small error (0.10m) in applying the shift to a point common to two polygons caused a serious increase in the size of the difference between the result and the reference data (1.06m, Table 9.3, polygon 438) while the precision estimators gave no indication that something was wrong. The cause of the magnification of the error was that the polygons interfaced

along a very short side, and the error as a percentage of the side was large. When this same error was multiplied along the length of the polygon the one metre error occurred. This was as a result of the incorrect transference of the scale from one polygon to the next, along the shortest side of each of the polygons.

A solution to both the scale and common point or connectivity problem was to adapt the mathematical model to allow only one scale and orientation correction per survey record. This ensured a link across polygons from the same survey record which had only one point common with another polygon, thereby eliminating the singularities or instabilities in the data. It also ensured that no error in a single side or polygon was dominant in establishing the scale and orientation correction, because these were now common to a number of polygons.

This mathematical model requires two points common to polygons from different survey records and one point common to polygons from the same survey record.

The data were reprocessed after adaption of the least squares network program and the results were exactly as expected. See Appendix G titled "Results: Test Area One One Scale and Orientation Correction per Survey Record". The connectivity and scale problems were solved.

The data for test area two had numerous errors and geometric oddities which caused the results to be inaccurate at first. There was, however, always an indication of the problem in the precision estimators, and so these were detected and eliminated. The final step of the process was to remove the trigonometrical beacons from an Lo polygon (GV1352) which lay in the middle and adjacent to one of the Goldfields diagrams. The results were within 0.08m (Y difference) and 0.18m (X difference) of the reference data. The adjustment was therefore considered a success.

A brief questionnaire (Appendix I) was completed by those responsible for the calculation of the geometric base for the GIS in Transnet's Johannesburg office. The data are potentially biased because of the small sample (three individuals), and their relationship to the author (Appendix J). In the absence of other data they are presented.

Their responses indicated that the least squares process was "Much easier" to operate and "Much quicker" to produce results when compared with the manual process. It was also their opinion that the production of precision estimators in the manual process was somewhere between very difficult and impossible and that, were such statistics produced, they would be very unreliable. Graphical methods, such as error figures, are certainly technically possible, but the time required for such solutions makes them practically unfeasible.

CONCLUSIONS

The mathematical model using one scale and orientation correction per survey record, when applied to the test data, yields correct coordinates and reliable precision estimators under sufficient circumstances without the need for data manipulation, to declare it a reliable method for partially automating the determination of unique point coordinates from diverse cadastral data.

The process is simpler than current manual methods, and the process is quicker than current manual methods.

The mathematical model using one scale and orientation correction per survey record, when applied to a global data set, will probably yield correct coordinates and reliable precision estimators. Certain circumstances may require data manipulation. It can therefore not be declared a reliable method for partially automating the determination of unique point coordinates from global diverse cadastral data without certain reservations, mentioned for investigation under the recommendations.

RECOMMENDATIONS

With respect to programming changes it is recommended that:

- An interactive data editor be created to enable manipulation of data during the different phases of processing.
- Data file selection be made interactive.

The following recommendations relate to general issues:

- The latest surveyed Lo value for a point should be adopted.
- All Lo points should be adjusted to the latest list of co-ordinates for trigonometrical beacons.
- Original polygon data should be stored as attributes of the polygon.

The areas requiring investigation are:

- The weight model for two reasons. The first reason is that σ_0 aposteriori was repeatedly in the vicinity of 0.01 while σ_0 apriori was fixed at 0.05. This is an indication that the observations are more precise than initially expected. This may or may not be true and needs to be examined. The second reason is that it is known that the one pre-85 survey was executed with an EDM and not a tape as assumed.
- The cause of geometric singularities or instabilities, which it should be possible to detect with a suitable algorithm in a program.

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GLOSSARY, ABBREVIATIONS AND SYMBOLS

Accuracy	in this thesis means the absolute correctness of a result within the context of the whole, or external accuracy, as opposed to precision.
Apriori	means before the adjustment.
Aposteriori	means after the adjustment.
Average error	is the sum of the errors divided by the number of errors
CAD	computer aided draughting.
Cadastral System	a system for recording the extent, ownership and value of land for taxation purposes (Fowler,1964:166)
Comparison plan	A plan on which data from diagrams in the form of angles, distances and point descriptions are compared with the same information derived from survey, with the purpose of confirming surveyed points.
Connection	in the context of a land surveyor's diagram, means data between an apex of the polygon represented on the diagram, and a point from some other polygon.
Data consistency	a test which calculates the misclosure of the recorded directions/angles and sides/distances of a polygon with the coordinates defining its apices. The test usually calculates an area for the polygon which may also be compared with that defined as an attribute of the polygon on the land surveyors diagram or in a cadastral database.
Datum	in relation to a coordinate system is some point used to define a particular value which is held fixed and relative to which all other points are determined. An

example is the Greenwich meridian which globally defines the line of zero longitude.

dP see positional error.

dX,dY standard deviation, or error in X and Y coordinates.

ECL Transnet's Electronic Coordinate List.

EDM Electronic Distance Measurement.

Expropriation a right in law to take ownership of property before formal transfer, which cannot be refused.

GIS Geographic Information System.

Goldfields system a number of different local coordinate systems in the former Transvaal. See Section 2.4.5.

Join the calculation of the direction and distance between two points defined by coordinates. The opposite to the calculation of a polar.

Mathematical
model the mathematical formulae used to define a situation.

Overshoot an intersecting line terminal which lies beyond the line with which it intersects, instead of on it.

Polar the calculation of the coordinates of a point using the direction and distance from another point. The opposite to the calculation of a join.

Polygon	an area enclosed by a figure of three or more sides.
Positional error	dP , is the square root of the sum of the squares of the errors in X and Y, dX and dY .
Precision	in this thesis means the relative correctness of a result in relation to surrounding or connected results, as opposed to Accuracy. Inner accuracy.
Pseudo observations	are data used as observations which are in fact not direct observations, but are derived from other observations or data. See section 7.1.
REGIS	a proprietary GIS package.
s	is a symbol representing a distance observation.
Sides and angles	the length of the sides of a polygon and the internal angles of its apices.
Sides and directions	the length and the direction, within a defined coordinate system, of the sides of a polygon.
Sliver polygon	in the context of a GIS, usually a long thin polygon accidentally created by the geometric inconsistencies of the data, which does not represent any real feature. See section 1.1.3.
Survey Act	in South Africa refers to Act No 9 of 1927.
Survey record	a comprehensive record of how a survey was carried out, including a co-ordinate list, a report, the field book, a working plan showing all points and distances measured, a complete set of all calculations relating to the survey including the confirmation of found beacons and the recalculation of missing points.

Survey system	A system consisting of a unit of measure (metres, cape feet, etc), a set of coordinate axes, and reference points in all but some local systems, enabling connection to the system.
<i>t</i>	is a symbol representing a direction observation.
TAGIS	Transvaal Association for Geographic Information Systems
Topology	in relation to a GIS means the relationship between features, such as adjacency, containment etc.
Traverse	a calculation mechanism consisting of polars executed sequentially from point to point.
Triangulation	A method of determining coordinates based on the measurement of angles or directions.
Trigonometrical beacons	in South Africa these are marks (usually a black vein in a white pillar placed in a prominent position), which have been coordinated on the national survey system (The Lo system).
Trig	is a commonly used abbreviation for trigonometrical beacons as described above.
TSM	is a Town Survey Mark coordinated on the Lo system. Usually a peg in a protective covering in the ground.
TSTLO	is the filename of the reference data set. The 'LO' in the name refers to the Lo coordinate system.
TSTLONR	is the filename of the data set which simulates a local system, because of the

their being 'No Reference'.

TSTLOOR	is the filename of the reference data set used when processing with the second or amended mathematical model. The 'OR' in the name implies original, or unaltered.
TSTLOS	is the name of the data set which simulates disparate local coordinate systems because of the addition of 'Shifts' added to the coordinates of certain polygons.
Undershoot	an intersecting line terminal which lies before the line with which it should intersect, instead of on it.
X, Y	are the names of the orthogonal coordinate system axes, conventionally also used as a variables for a general reference to coordinates in the system .
σ_0	the standard deviation of the observation of unit weight.

CHAPTER ONE
INTRODUCTION

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1.1 INTRODUCTION

Transnet possesses approximately 60000 land parcels covering the whole of the South African rail network. These parcels will form the base map in a corporate Geographic Information System (GIS) which will be used for property administration and general railroad operations management. Accuracy requirements are high, and unique coordinate values have to be determined for each parcel corner point before data are input into the GIS. A railway property portfolio's parcels are characteristically configured in a series of long narrow strips. The solution therefore has to cope with poor geometry.

Methods which are economical in terms of time and cost and which produce acceptable levels of precision in the data are required. This thesis describes the development of a mathematical model using the least squares parametric network adjustment method as a means of addressing the problem.

1.2 THE NEED FOR THE CADASTRE IN A GIS

Ownership or control is the determinant of decision making for resources in an economy. For land and land related features, such as buildings, utilities and natural resources, this is determined by the ownership or control of the land on which such features are located.

A *Cadastral System* (Fowler, 1966:166) records details of the position, extent, ownership and value of land or rights in land. A cadastral system may, however, include a combination of other elements, such as *land use* and *physical features*. It could then be described as a multipurpose cadastre as opposed to either a purely legal or a purely fiscal cadastre as described by Fowler.

For a Geographic Information System (GIS) to support decisions relating to land-based features, information detailing ownership and control of the land-based features must be a part of the GIS. This implies that the geometric or spatial base of the cadastre must be in the GIS,

and that it logically forms the base of the GIS onto which other features are superimposed.

"Land surveyors' diagrams" or "cadastral diagrams" record the data defining the component polygons of the cadastre.

1.3 CHARACTERISTICS OF THE SOURCE DATA

The data systems used on cadastral diagrams to define the geometry of polygons have changed over time. Diagrams in existence at the time of a change are not updated or transformed to the new system, and are therefore not directly compatible with earlier or later systems. The degree of difference varies and the processes required to achieve compatibility and the quality of the results vary.

In South Africa the units of distance have included Rhymland Roods, Rhymland feet, English feet and inches, Cape Roods, Cape feet and inches, chains and currently metres (Koetsier, 1969: viiii). A variety of projections have been used, from the local plane to the transverse Mercator. Some have been corrected for height above sea level and some not. Local coordinate systems specific to a polygon and systems of limited extent, such as the Goldfields system, have also been used.

Multiple values exist for certain points. This is the result of different surveys defining adjacent polygons that have common points. The common points can be represented differently in each polygon according to the units, projection, coordinate system, ellipsoid or precision of the survey that defined them.

1.4 CHARACTERISTICS OF THE REQUIRED CADASTRAL DATABASE

1.4.1 Unique points

Conceptually the spatial base of the cadastre in a vector based GIS has a unique polygon

defining each parcel (erf, lot, holding, farm or portion). Adjacent parcels must share the same unique boundary lines and points. This prevents the creation of sliver polygons.

Sliver polygons corrupt the spatial/geometric part of the cadastral database by the spurious introduction of polygons representing nonexistent features. Once classified in a GIS, this in turn corrupts the topology in the GIS database and will cause errors in answering certain queries. In other words, sliver polygons cause the data to become unreliable. The system is then not able to yield correct answers, rendering it untrustworthy. An untrustworthy system cannot fulfill its role of decision support through information storage, manipulation and analysis, and is of questionable value.

The shaded area in Figure 1.1 below shows a sliver polygon arising from multiple values for the point common to the polygons representing erven 1 and 2 ('B1' and 'B2'). The point 'B?' represents the undetermined unique point required to represent the point common to both polygons. The development of a method for the partial automation of the determination of a coordinate for this unique point is the purpose of this thesis.

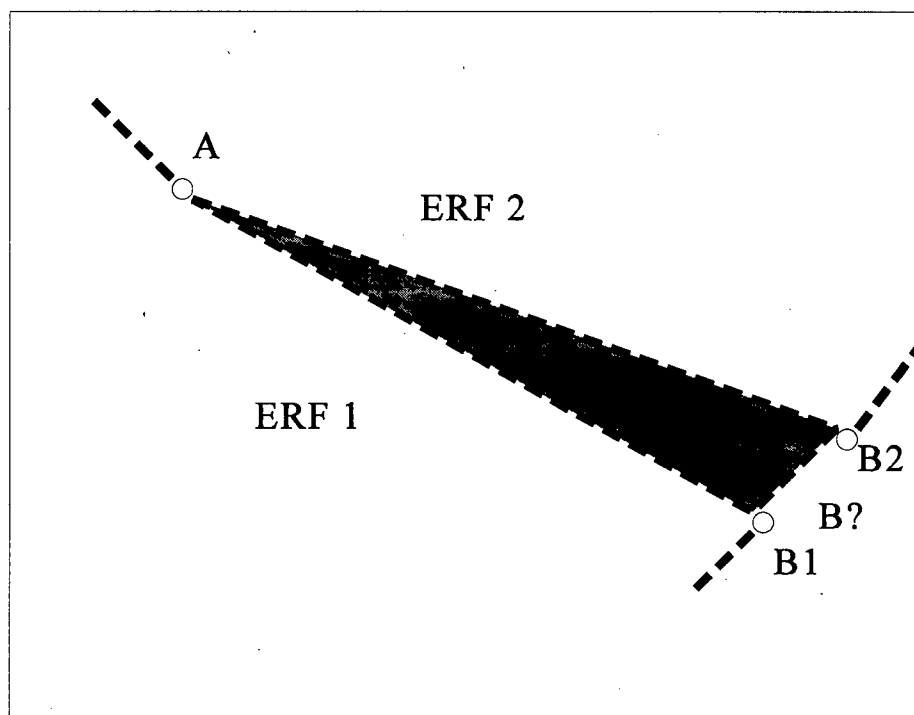


FIGURE 1.1 SLIVER POLYGON

1.4.2 Coordinate system

A single coordinate system must be selected for the spatial cadastral database to integrate all the component polygons. South Africa currently uses the Lo system. This is a country wide system of two-degree-wide panels, with the origin at the intersection of the equator and the odd numbered meridians. The system is a transverse Mercator projection of the Clarke 1880 modified ellipsoid. (See 2.4.1 - 2.4.4). There is a strong likelihood of a change to the WGS84 ellipsoid in the near future. (See 2.4.10).

1.4.3 Precision

Precisions of less than ten centimetres in urban areas, and less than thirty centimetres in rural areas are required. This requirement is based on a knowledge of the legal precision requirements for cadastral surveys, and an experience-based intuitive insight into the precision that can be obtained from cadastral data.

The expected precision is qualified by the knowledge that different eras have produced different levels of precision. Some earlier data are less precise (one metre or more) and some newer data more precise (sub-centimetre). This is because instruments, techniques and control networks have become more precise over time. Ultimately a precision of not less than twice the assumed precision of the source data is acceptable. This principle is in line with the requirements of the survey regulations in terms of the Land Survey Act, No 9 of 1927.

The functionality currently envisaged for a GIS with a cadastral base does not generally require a higher precision than mentioned in the first paragraph of this section. If, however, system precision were to be upgraded over time, then new uses demanding higher precision from the GIS could be developed. The unique point coordinate determination should therefore achieve the maximum precision allowed by the source data.

1.4.4 Integrating the cadastre and other features

One purpose of establishing the cadastre in a GIS is to make possible the integration of other features (such as buildings and services) so that they may be managed effectively and efficiently.

The data defining such features are usually captured by ground survey or photogrammetry. Both these techniques usually reference their coordinate systems to the latest published list of trigonometrical beacons (trig). The integration of these features with the cadastre is therefore best achieved by having the cadastre defined, as accurately as the source data will allow, relative to the same trig. (See 3.3 and 5.1).

1.5 USER-INTERACTIVE METHODS OF DATA CAPTURE

Two methods of capturing digital cadastral data are traditionally used. These are digitising from compilation plans of cadastral data, and calculation using the data on cadastral diagrams.

Digitising from compilation plans yields precisions dependant on the scale of the plan being digitised and operator pointing error. Using plans at scales of 1:1000, a typical operator pointing error of 0.0005m to 0.001m (half a mm to one mm) yields errors of 0.5m to 1.0m. The method is rejected because this is beyond the defined precision requirements.

Helmert transformation, polar and traverse are typical calculation techniques. These require a large degree of user interaction. This demands operator skill because of the decision-making required about which technique to use with which data, and assessing the acceptability of the results.

One disadvantage of high levels of user-interaction is a poor time/cost to output ratio. There are also low levels of consistency in output, because of low levels of consistency in operator decisions. This is a result of operator preferences, and results in biased data. Precision

estimation is often non-existent, or is only printed and not stored with the coordinate determinations. Precision estimation is therefore effectively 'lost' to the database and the GIS.

Some manually interactive methods result in the permanent entrenchment of errors or imprecision in the system. This is because there is usually no indication of how a point has been fixed, nor of the precision associated with such a fix. Subsequent point determinations are then built onto the erroneous or imprecise foundation. Should an error be discovered it may often be impossible to determine which points are affected, and hence impossible to remove the effect of the error from the system.

1.6 THE RESEARCH PROBLEM

The essence of the problem is to determine a unique value for a point from multiple, potentially conflicting data sources.

The research question is:

"Can a mathematical model be defined using the parametric least squares survey network adjustment method to determine unique point values from cadastral data?"

1.7 CONCEPTUAL SOLUTION

The concept underlying the mathematical model rests on three foundations.

First, the directions and distances of polygon sides are obtained or derived from cadastral diagrams and used as pseudo observations.

Second, the apices of polygons surveyed on the national coordinate system (Lo) are held fixed in the adjustment, thereby defining a single coordinate system.

Third, the fixed points and pseudo observations are used in a least squares network adjustment, to determine coordinates for those unique points not surveyed on Lo and not held fixed in the adjustment.

1.8 THE OBJECTIVES OF THE STUDY

The objectives of the study are:

- To develop a mathematical model for a least squares network adjustment program, based on the principles described in the conceptual solution. (See 1.7).
- To test the methodology on the selected test data sets to see if:
 - i it yields correct coordinates.
 - ii it produces reliable precision estimators.
 - iii any circumstances under which it may fail can be identified, explained and resolved.
 - iv there are significant levels of correlation in the results.
- to gauge the usefulness of the method compared with alternatives with respect to:
 - i its simplicity of operation.
 - ii its speed of operation.

1.9 CONTRIBUTION OF THE STUDY

The impact of the problem is described followed by an explanation of both the scientific and practical contributions of the study.

1.9.1 Impact of the problem

Transnet's cadastral database, which has been used in the case study, consists of approximately

80 000 entities (owned, alienated and adjacent land parcels as well as servitudes and other rights). The study suggests that a saving of 25 000 man-hours, or 3125 man-days or 156 man-months or 14 man-years is possible. The data from which these figures are derived may be highly biased. Even if these figures are wrong by a factor of 10, which is highly unlikely, the time savings are still significant.

Any organisation in South Africa, or anywhere in the world where a similar cadastral system exists, will face the same kinds of problems in the creation of a digital spatial cadastral database.

1.9.2 Scientific contribution

The scientific contribution of the study is, first, the development of a method, via the mathematical model, of combining cadastral data that has not been encountered to date.

A second contribution is the testing of the method in instances of geometry defined by long chains of polygons, which is generally considered weak geometry from a survey perspective.

1.9.3 Practical contribution

Automating the process of combining data and reconciling inconsistencies speeds up the creation of a digital spatial cadastral database, compared with user-interactive methods. This could have significant time/cost saving implications. The degree of saving depends on the efficiency of the user-interactive method which the automated method replaces.

Reducing the number of decisions by operators to resolve inconsistencies will ensure a greater level of consistency in the creation of such a data base. It simplifies the complete removal of detected errors from the system, because the method and logic behind the creation of each point are known. The data are dealt with in distinct geometric blocks, and it is therefore simple and quick to reprocess an entire block.

Determining estimates of precision for the fixes of the points supplies a more comprehensive and accurate picture of the reliability of a database, than is currently obtained from the assorted mixture of methods used in the user-interactive process.

All these factors lead to higher levels of reliability in the data. This results in greater confidence in the data and therefore the GIS.

1.10 THE TRANSNET CASE STUDY

The author is an employee of Transnet's Land Surveying Services department, which has been tasked with the establishment of an organisation-wide multipurpose GIS. All other data sets will be mapped onto the cadastre, which was chosen as the base data set.

Transnet is the owner and operator of, amongst others, the national railway system. Railway property consists by nature of long, narrow chains of polygons. Much of Transnet's property has been acquired piecemeal over time resulting in many irregular internal cadastral boundaries. Internal cadastral boundaries are those where Transnet is the owner of the land on either side of the boundary, whereas external boundaries are those between Transnet and its neighbours.

An intense program of rationalising the use of Transnet's assets has resulted in the need to sell unused land. Registering different portions in the names of the various units of Transnet enables accurate accounting of costs, income and return on capital. This will hopefully ensure better management of the property portfolio and the related assets.

Buildings and services were sited without any consideration being given to internal cadastral boundaries, and often with little concern for the exact position of the external boundaries. It is a principle in South African law that a fixed structure cannot be owned separately from the land, except in the case of Sectional Title. Land and structures are seen as inseparable physically, and therefore also in law. Transnet follows the same policy with respect to its own assets and land, in as far as internal division of fixed assets is concerned. Hence a building that straddles an internal boundary between two divisions needs to be identified and the boundaries

redefined.

The GIS for which the data are being prepared is REGIS, and the CAD system used in the data capture phase is REMAP (previously Ultimate Cad - Mapping). A variety of proprietary and in-house survey calculation packages and tools have been used in the manual (user-interactive) data capture process. The intention is that the methodology developed here will replace some of these more labour intensive methods.

1.11 THE RESEARCH METHOD

1.11.1 Mathematical model development

Two mathematical models have been developed and tested. Both use the direction and distance between the polygon apices as observations. The first allows one scale and orientation correction per polygon. The second model allows one scale and orientation per survey record making these corrections common to all the polygons grouped in a survey record.

1.11.2 Mathematical model testing - Coordinate accuracy

To test if the method yields correct coordinates a control data set was used. Results were assessed by differencing the calculated coordinates and the control set. A difference of less than twice the assumed precision of the source data was defined as acceptable.

1.11.3 Mathematical model testing - Precision estimation

Precision estimation was examined by assessing if the trend in the accuracy of the results was reflected in the precision estimators for the results. This was done simply by examining the tabulated meaned accuracy comparison and precision results for each adjusted polygon.

1.11.4 Mathematical model testing - Correlation

The correlation between the determined unknowns is reflected in a spreadsheet in Appendix K. It was derived from a single test-area-two data set. The standard method of dividing the off diagonal terms of the $(A^t w A)^{-1}$ matrix by the root of the appropriate two main diagonal terms was used.

1.11.5 Usefulness testing

A simple questionnaire was used to examine user perceptions as an indicator of usefulness.

1.11.6 Practical procedures and decisions

For the research objectives to be achieved several practical decisions had to be made and a number of procedures developed.

One practical decision related to the correction of polygon apex coordinates because of changes in the trig coordinates. This decision is discussed in Sections 1.13.3 "*REFERENCE POINT ADJUSTMENT*", 2.4.2 and 3.3. Another practical decision related to the alignment of beacons. This decision is discussed under Sections 1.13.1 "*ALIGNMENT OF BEACONS*" and 3.2.

The applicability of legal cadastral constraints on the creation of a digital cadastral database is discussed under section 3.1. Section 3.4 highlights the difficulties of distinguishing between points that are close together and different values for the same point.

Practical procedures included the writing of the programs and extraction of the data sets from Transnet's cadastral database.

1.12 THESIS LAYOUT

General theory and investigation into alternative techniques are discussed in chapter two. Chapter three discusses some practical issues such as adjustment to the latest trig list, alignment of beacons, and the difficulties associated with different points that lie close together.

Chapter four examines Transnet's data capture process and describes the program used to extract the data from the database, and prepare it for the next phase of processing in the method. This is followed by a description of the programming for determining which points represent the same point (referencing), and adjustment of Lo polygons for changes in the trig coordinates in Chapter five. Chapter six describes the form of the observation equations, the weight model, the formation of the matrices and vectors and precision measures and checks. Support programs that facilitate the operation of the core programs are described in chapter seven.

Chapter eight describes the data and the results. Chapter nine describes conclusions based on the results. In closing, Chapter ten makes recommendations for use and research.

1.13 SCOPE, ASSUMPTIONS AND LIMITATIONS

1.13.1 Scope

The aim of the study is not to develop a complete operational program that takes account of all possible data and situational variations. It is to test if the defined mathematical model using the least squares method to partially automate the determination of unique point coordinates is fundamentally viable. Its speed of operation and ease of use then determine if it is a feasible alternative to other methods of data capture.

Because of the nature of the data used, the least squares method has been employed in a pragmatic non-rigorous way that produces results.

DATA SOURCES

Discussion has been limited to the South African cadastral system and to techniques and data sets available and relevant to the Johannesburg office of Transnet's Land Surveying Services Department.

Test data have been limited to two areas where accurately surveyed coordinates on the Lo system are available to serve as control data. The test data have been manipulated into several different sets to simulate the characteristics of some of the data systems available. (See Chapter 9).

The data in test area one (see Section 9.1) are all from a group of related survey records and form a highly consistent, highly precise set, which is not typical of all data sets. It does, however, form a good control set for testing the rigour of the results produced by the method.

Usefulness gauging has been restricted to questioning those technicians in the Johannesburg office of Transnet's Land Surveying Services department who have been directly involved in the creation of the digital cadastral database. They are presumed to be the only 'unbiased' people with sufficient insight to be able to make comparative estimations. This is because only they and the author have been exposed to both the existing process and the method in this thesis.

TOPOLOGY

No attempt has been made to create a topologically structured database. There are two reasons for this.

The first is that this is done automatically by the GIS for which the spatial data base in the study is being created. All that the GIS requires is that "clean" data are submitted to it for classification. Clean data are data in which there are unique points defining each polygon apex, and in which there are no overshoots or undershoots. (See "*ALIGNMENT OF BEACONS*" below).

The second reason is that in order to create correct topology the data inconsistencies must

already have been resolved. The resolution of the data inconsistencies to create unique points remains the limiting focus of this study. The possibility of utilising the topology created by the GIS with the source data in a "second iteration" to improve the precision of the results is a concept worthy of investigation. This will not be considered in this thesis other than as a defined limitation.

DATA MANAGEMENT

Spatial data base design and management have only been addressed in passing comment, as it does not affect the core aim of this study. The format of such a database is not relevant to the resolution of the data inconsistencies. This is especially true when a topological structure is not required in the spatial database prior to classification in the GIS, as in this study.

ITERATION

No iteration has been used in the test version of the program, and will only be added if the program is converted to an operational version. The reasons for this are explained more fully in Chapter 7.

ALIGNMENT OF BEACONS

No attempt at aligning beacons between terminals has been made for the reasons explained in Section 4.3. There are instances where misaligned beacons cause overshoots or undershoots resulting in sliver polygons. These are eliminated by reconnecting the polygon boundary lines in the pre GIS CAD phase. No attempt at eliminating or resolving these situations has been made. This is because these situations do not arise from multiple representations for a point. Special techniques would be required to detect them. Their resolution was not considered significant to testing the fundamental viability of the method.

1.13.2 Assumptions

PRECISION

The mathematical model can be considered valid only if it produces results at an acceptable

level of precision using valid tests on valid data. A result within twice the defined level of precision for the input data will be taken as acceptable, in line with standard practise in the survey regulations. This is based on the assumption that adjustment of a survey is unlikely to yield more precise results than the original survey that will itself have been adjusted and checked by the land surveyor performing it. The adjustment should, however, not cause a significant deterioration in the quality of the data, as reflected in the precision estimators. Some inconsistencies between adjacent polygons may cause a marginal deterioration in some point precision estimators. If this is less than twice the assumed precision of the source data then it is not considered significant.

LATEST LO VALUE IS MOST ACCURATE

When a point is represented on more than one Lo referenced polygon, the latest dated point is assumed to be the most accurate and precise. This assumption is based on the fact that instrument precision, especially that of distance measuring equipment, has improved with time, albeit marginally. It is difficult to conceive of a network adjustment producing results of greater accuracy than the original Lo based surveys did. The least squares network adjustment is used to reconcile disparate non-Lo data sets, which must be manipulated to fit the existing Lo data. Points from Lo referenced polygons are shifted for changes in the trigonometrical beacon coordinates, and then held fixed.

OBSERVATIONS ARE UNCORRELATED

To create the weight model in as simple a fashion as possible, it was assumed that the pseudo observations are not correlated. This is not true. Observations to and from a point are derived from the same coordinate, and are therefore correlated. The significance of this correlation is assumed to be low, and its effect on the results has therefore not been examined.

1.13.3 Limitations

WEIGHT MODEL

The weight model has been kept as simple as possible in order not to obscure the fundamental quality of the results with extraneous issues. This is the reason for selecting a highly consistent,

uniform set of data such as that in test area one. Different eras are known to have produced different precisions in point fixes. The only distinction made is between pre EDM era surveys and EDM era surveys.

The weight model assigns equal weight to observations between points from the same polygon irrespective of the distance between them. This is based on the lack of knowledge of how precisely any particular point is fixed. Each point is therefore presumed to have been fixed with the same precision. This ignores the fact that the relative precision of points is usually proportional to the distance between them. This needs to be investigated and the model may need amendment. It was, however, not considered of great significance in terms of the aims of the thesis.

TEST DATA GEOMETRY

Both test data sets consist of long narrow chains of polygons typical of the geometry of Transnet's property portfolio. This is a weak geometrical configuration for a network. Ultimately the geometry and the data define the limits of precision that any method will yield. If the method had yielded poor results, then it would have to have been proven, via alternative methods, that the data could be used to produce good results before the method could be considered to have failed

The method has not been tested on a well-formed network of polygons. The assumption is that if it works on weak geometry it will work on well-formed geometry. This however must still be tested.

The weak geometry has highlighted certain geometric situations where the method can either not produce a result, or produces unreliable results. (See 9.1.3). Unreliable results are worse than no results, especially if there is no indication that the results are unreliable. The geometric situations producing unreliable results are identifiable and correctable by a skilled person who understands the nature of the problem. They may also be detectable by programme, but no algorithm has been developed or tested for this purpose.

REFERENCE POINT ADJUSTMENT

There are numerous methods of adjusting the coordinates of polygons as a result of changes in the values of trigonometrical beacons. (See 2.4.2). For this study a shift calculated from the mean difference between data and current published values was used.

TEST DATA FORMAT

The data used in the study were extracted from Transnet's ECL database. (See 4.1). These data have unfortunately been stored in the ECL as coordinates only, and do not include the length or direction of sides, or angles of the polygon apices, as recorded on some cadastral diagrams.

This made some simple and common data validation checks difficult. A simple example is the checking of the consistency of the data for the sides and directions/angles with the coordinate data. Another is comparison of the area derived from the data with that recorded on the diagram and stored in the alpha-numeric database. None of these checks have been done or programmed. Coordinates were checked by manual comparison with the source data. This was considered adequate for the purposes of the study because the initial set of coordinates was used as the control set, and manually manipulated variations of these sets were used to simulate the different data systems. The ability of the method to recreate the reference data from the manipulated data is the measure of its success, irrespective of whether the reference set was truly correct or not. The tests could have been run on purely simulated data, but it was convenient to use data already captured. This resulted in the added benefit of the identification of problems with existing data and methods.

SCALE AND PROJECTION ENLARGEMENT

Provisional Lo coordinates have been used to calculate the pseudo observations. This means that these factors have been built into the derived pseudo observations and need not be built into the model for the purposes of the thesis.

NON-COINCIDENT POINTS

No method of distinguishing between distinct points that lie close together and multiple representations of the same point has been developed. (See 3.4).

APPLICABILITY TO OTHER DATA SETS

The aim of the study is to determine the fundamental viability of the mathematical model by testing it on data sets with well-established precision and accuracy. Global applicability to other data sets will still need to be established.

The principles in this paper should however be generally applicable to any other similar cadastral based GIS in spite of the defined limitations. Special techniques may need to be developed to account for unusual features in some systems.

QUESTIONNAIRE BIAS

The questionnaire used to gauge user perceptions of usefulness is potentially highly biased. It therefore only serves as a vague indicator of a local trend, and not an establisher of global highly probable trends. The results are presented because of the lack of other data.

1.14 CHAPTER SUMMARY

For a Geographic Information System (GIS) to support decisions relating to land-based features, information detailing ownership and control of the features must be a part of the GIS. This implies that the geometric or spatial base of the cadastre must be in the GIS, and that it logically forms the base of the GIS onto which other features are superimposed.

Conceptually the spatial base has a unique polygon defining each cadastral parcel. Adjacent parcels share the same unique boundary lines and points. This prevents the creation of sliver polygons that corrupt the GIS database, and cause it to be unreliable in answering certain queries.

Two user-interactive methods of capturing digital cadastral data are traditionally used. These are digitising from compilation plans of cadastral data, and calculation using the data on cadastral diagrams. Digitising is rejected because it results in errors beyond the defined precision requirements. Calculation generally requires a large degree of skilled user interaction.

The research question is, "Can a mathematical model be defined using the parametric least squares survey network adjustment method to determine unique point values from cadastral data?" The concept underlying the solution rests on three foundations. First, the directions and distances of polygon sides are obtained or derived from cadastral diagrams and used as pseudo observations. Second, the apices of polygons surveyed on the national coordinate system (Lo) are held fixed in the adjustment, thereby defining a single coordinate system. Third, the fixed points and pseudo observations are used in a least squares network adjustment, to determine coordinates for those unique points not surveyed on Lo and not held fixed in the adjustment.

The aim of the study is not to develop a complete operational program that takes account of all possible data and situational variations. It is to test if the defined mathematical model using the least squares method to partially automate the determination of unique point coordinates is fundamentally viable. Its speed of operation and ease of use then determine if it is a feasible alternative to other methods of data capture.

CHAPTER TWO

GENERAL THEORY, ALTERNATIVE TECHNIQUES

FOR DETERMINING UNIQUE POINTS,

AND COORDINATE SYSTEMS

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2.1 LITERATURE ON TOPOLOGICAL DATABASES

There is a vast body of literature on the creation of topological databases. However, there is very little literature relevant to the problem addressed in this study. One explanation for this might be that very few countries have as substantial a part of their cadastral data on a national coordinate system as does South Africa.

The only remotely relevant piece of literature found was a half thesis by Francois De Wet (De Wet, 1995). The focus of his research and development was on data management. There are therefore only four pages devoted to the calculation of cadastral data (pages 44 - 48) which contain the statement "This phase should preferably be carried out by a person with experience in cadastral calculations", (De Wet, 1995: 44). This illustrates the typical reliance on skilled and experienced personnel to resolve inconsistencies in data. No further attention will therefore be devoted to his work for the purposes of this thesis. Readers should, however, note that some form of data management, such as that described by him, is essential in any cadastral data capture project.

2.2 ALTERNATIVE TECHNIQUES FOR DETERMINING UNIQUE POINTS

2.2.1 Techniques employed in other cadastral GIS's to solve the problem.

Several GIS consultants, and the persons responsible for establishing or the administration of various cadastral based GIS were consulted to find out what technique or philosophy they employed to solve this problem.

There was a general if vague consensus on the approach adopted. Those consulted were:

Land surveyors:	Vosloo H, Milne D, Newey W.
GIS consultants:	Zcese Z, Moreau L.
GIS vendors:	Mulder C, Kotze J (also a land surveyor).

None of the above could produce a document defining the philosophy or the procedure for

selecting a point, or were aware of any paper or documentation on the topic. The consensus among the land surveyors was that the highest accuracy requirement was about 0.10m, and most were happy with less precise results of up to 0.30m for old or rural data. Standard survey procedures, such as traversing between block corners and acceptance of results with misclosures of better than 0.10m are usual. Johannesburg City Council and Eskom (Vosloo H), Cape Town City Council (Milne D) and Rand Water Board (Newey W) all reported that they used this, or similar, methods.

A municipal GIS is typically of a limited extent, and well covered with township surveys that are of a high order of precision. This makes a precision of 0.10m more easily achievable than in rural areas.

In contrast, Transnet's property covers the entire country, and varies greatly in format, coordinate system and precision. The number and complexity of circumstances that require decisions by an operator working on such data are therefore more than those usually encountered in a municipal situation. Invariably, survey inconsistencies have been absorbed by the railway line that, for example, often lies as a farm portion between two townships surveyed on different datums. Often the railway line is the only remaining extent of a farm, the deducted portions of which have been surveyed on different systems. This leaves a narrow strip of the remainder without an integrated survey.

2.2.2 Alternative user-interactive techniques

As mentioned in Section 1.4 there are basically two approaches to creating a digital cadastral data base, namely digitising and calculation. Digitising was rejected as a technique because of the low precisions obtained.

Calculation techniques are limited by the extremes of a continuum of the level of automation. Most organisations are using manually intensive methods. This is probably because no successful automated methods for combining cadastral data from diverse sources are known to them.

Manual methods may involve computers. Typically a traverse is run from one known point to another. Misclosure is given as an estimate of precision. Such a precision estimate seldom takes account of the precision of the start and end points of the traverse, and makes no provision for storing the precision estimate with the determined coordinate for the point. This information is potentially very valuable in assessing the reliability of certain analyses that the GIS may perform.

It is usual to use only one side of a polygon in traversing from one point to another so that the shortest route is used. The traverse is then expanded from the adjusted points to include the rest of each polygon, and any additional polygons that may lie indirectly along the route. This means that any adjustment of a traverse is usually on the basis of a limited percentage of the available data. The method in this thesis seeks to use all available data in adjusting between known points and to provide precision estimators for storage with each point.

There are many calculation techniques, such as the Helmert transformation and intersection that may be employed. They generally all suffer from the same shortcomings as far as the creation of a digital cadastral database is concerned. One shortcoming is that only a part of the available cadastral data is used. Other shortcomings are in most cases the failure to take account of the precision of points on which the techniques are based, and the failure to calculate or store precision estimators.

2.3 LEAST SQUARES - AN OVERVIEW

2.3.1 A conceptual description

Least squares is a technique for solving for the unknowns in an overdetermined solution. In other words, when there are more observations than unknowns, then there are liable to be inconsistencies. Least squares accommodates these inconsistencies by allowing a residual error to be associated with each observation. The values for the unknowns are then determined so that the sum of the squares of the residuals is a minimum.

2.3.2 The basic formulae

The parametric adjustment technique is commonly used in survey situations. Here the observations (distance and direction in this study) are written in terms of the unknowns (coordinates, scale factor and orientation correction in this study) and is termed the mathematical or functional model.

In this study there are no genuine observations. The derived observations are therefore termed pseudo observations. The way in which the pseudo observations are defined creates the mathematical model. The purpose of this study is to define the observations and their relationship to the unknowns in such a way that acceptable results are produced. See Chapter 7 for a more detailed description of the mathematical model employed in this thesis.

One equation is written for every observation. These are then linearised (if nonlinear) and the result is n simultaneous equations in m unknowns. If n is greater than m then the equations are translated into matrices and vectors and the unknowns resolved for in the following way.

$v = AX + f$ is the basic equation, where v is a vector of residuals, A is the matrix of coefficients of the unknowns, X is the vector of unknowns, and f a vector of constant terms.

$X = (X) + x$, where (X) is an initial approximation of the unknowns and x is a vector of adjustments to the approximate unknowns. The vector f is made up of constant terms from the linearisation, represented by the approximation to the unknowns (X) , and the observation.

A weight matrix w may be introduced, which reflects the varying precisions of the different observations, thereby altering the influence of the different observations in the solution.

The least squares principle demands that $v' w v$ is a minimum, where w is the weight matrix.

It can be shown that $x = (A'wA)^{-1} A'wf$ gives the desired result. The reader is referred to a standard text on least squares such as "*Observations and Least Squares*" (Mikhail, 1976) for

a more detailed explanation.

2.4 SURVEY SYSTEMS AND DATA SOURCES

Data on diagrams in the former Southern Transvaal are available in several different survey systems: These are listed below with the characteristics of each system most important to this discussion.

2.4.1 Lo system

This is the latest and most precise system available in South Africa on which all modern cadastral surveys are coordinated. Accordingly, it is the logical system for adoption as the reference coordinate system for the cadastral base of a GIS in South Africa. It has been selected as the reference system in this study. Examples of diagrams surveyed on the system are included as Appendix D3 (in cape feet) and D4 (in metres). The Lo system consists of several 2° wide panels based on a Gauss Conform (Transverse Mercator) projection. Figure 2.1 shows the origin of the Lo 27 coordinate system as a set of orthogonal axes labelled +X and +Y.

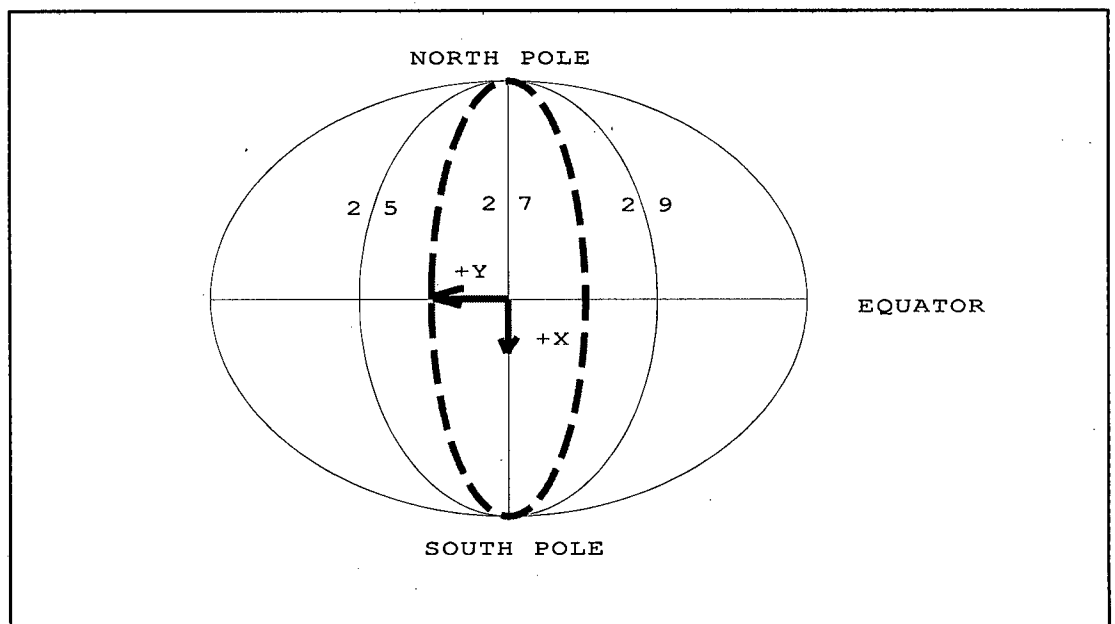


FIGURE 2.1: LO 27° COORDINATE SYSTEM

The origin of the coordinate system of each panel is at the intersection of the odd degree meridians and the equator. The system is clockwise with zero degrees South. Adjacent panels meet at the even numbered meridians.

2.4.2 Lo system - different local datums

The reference points for the South African national coordinate system, are based on observations between physical monuments, such as trigonometrical beacons and town survey marks (TSM's), from which coordinates for these beacons are calculated. In this thesis the term trigonometrical beacon is meant to include TSM's unless the context suggests otherwise.

Local surveys are then based on these calculated values. The coordinates assigned to the trigonometrical beacons are the only means of local connection to the datum, and therefore define the datum for these surveys. If an adjustment is made to the coordinates of these trigonometrical beacons, then the locally available reference to the datum for surveys based on such beacons is effectively altered. This is illustrated in Figure 2.2, where the effect of altering the coordinate values of trigonometrical beacons is shown to effectively move the origin relative to the local points.

Various adjustments to trigonometrical beacons have taken place over the years when errors have been corrected, or improved observations have been made. There are thus many different local datums for surveys that are nominally within the same survey system.

Land surveyors' diagrams approved by the Surveyor General form the backbone of the cadastral system. Since 1970 these diagrams allow for the reconciliation of different datums, by recording the coordinates of two trigonometrical beacons in the area of the survey on the diagram.

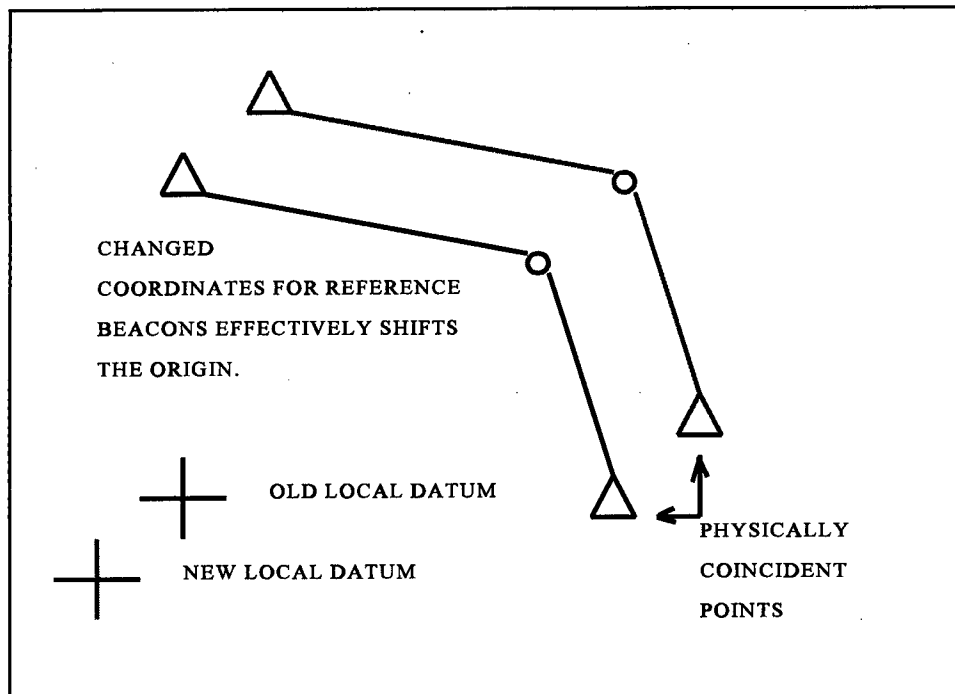


FIGURE 2.2: DIFFERENT DATUMS AFTER ADJUSTING TRIGONOMETRICAL BEACONS COORDINATES

This practice of recording the trigonometrical beacons was unfortunately not followed from the start of surveys on Lo in South Africa. This makes it difficult to determine what changes have taken place to the trigonometrical beacons in the area of a survey, and hence difficult to determine how to adjust the beacons shown on such diagrams.

A limitation to the value of the system is that the coordinates of any two trigonometrical beacons, appearing in the coordinate list of the survey, may be used on the diagram. This means that the quoted trigonometrical beacons are not necessarily those to which the survey is most directly linked in establishing the control for the survey. They may merely have been used for orientation.

Inconsistencies between adjacent trigonometrical beacons may be anywhere from 0.00m to 0.30m. This has been determined from ten GPS surveys of forty trig beacons in the former Transvaal by the author during mid 1996. The University of Cape Town has obtained similar results in the Western Cape (Merry, 1997: Telephone conversation). If trigonometrical beacons

quoted on a diagram have not been used directly in carrying in horizontal control for a cadastral survey and are used to adjust the coordinates of other beacons on such a diagram, then an error similar in size to that which the adjustment seeks to remove may, in fact, be introduced. It is, however, unusual to find errors in excess of 0.10m, and most errors are of the order of 0.05m. The method of reconciling adjacent surveys by adjusting polygon coordinates for changes in the trigonometrical beacons recorded on the diagrams is therefore considered valid and valuable.

There have also been situations where the recording of any coordinates on a diagram has not been compulsory. Sometimes these surveys have been based on the Lo system, and it is possible to obtain the coordinates used from the *Survey Record* filed in the Surveyor General's office. (See 2.4.9).

Many trigonometrical beacons have been destroyed over the years, and once destroyed, no new value for a beacon is calculated by the Chief Surveyor General's office, which is the authority responsible for this task. This means that the connection between an old survey and any new survey is effectively lost. There are many methods by which the coordinates for destroyed trigonometrical beacons may be calculated. This issue is relevant to a system implementation and will need to be solved at that stage. It will not affect the aims of this study and will therefore be ignored.

2.4.3 Lo system - different surveys.

Different surveys based on the same trigonometrical beacons will yield different values for the same point. These differences could be the result of:

- **Gross errors**
- **Random errors**, for example, as a result of instruments with differing accuracy capabilities, and the natural limit of human ability in pointing an instrument at a target.
- **Systematic errors**, such as different methods of observation and/or calculation.

2.4.4 Lo system - adjacent panels

Many surveys are done across the interface between the panels. It is usual to use the coordinate system that covers the bulk of the survey throughout. This is facilitated by the publication of coordinates for an overlapping band of beacons along the intersection of the panels in both coordinate systems.

A single point may have been coordinated in different surveys on different systems. Programs exist for the conversion of one panel's coordinates to those of the adjacent panel. The decision, concerning which points should be stored using either system, is therefore not a survey problem, but a philosophic or practical problem requiring a value judgement.

2.4.5 Goldfields system

Five different Goldfields Survey systems were defined in different areas of the Reef. Each has its own origin and marginally different orientation. Examples of Goldfields diagrams are included as Appendices D1 (cape rods) and D2 (cape feet). These systems were created in an era when instrumentation did not provide the same degree of accuracy as do modern instruments and techniques. This is especially true as far as distances are concerned. These systems are characterised by a low level of internal consistency, (see paragraph below).

Each system was based on observations to its own set of reference beacons. Most of these reference beacons have been destroyed, making the determination of a set of Goldfields to Lo transformation parameters difficult and inaccurate. Transforming from the Goldfields Systems to Lo can give transformation deviations of up to a metre over the relatively small distance of one kilometre (1/1000), and orientation swings are variable.

These inconsistencies have resulted in the Surveyor General Pretoria not accepting transformed Goldfields coordinates as the basis for reconstructing missing property beacons when these are to be replaced. The Surveyor General in Pretoria requires a reconstruction of the area based on the laborious process of comparing angles and distances on a comparison plan. A comparison

plan shows data from diagrams and data from surveyed coordinates of found property beacons to establish which are to be accepted. This is in contrast to his acceptance of a simple comparison of the coordinates on Lo based diagrams with surveyed coordinates to determine which found points are acceptable, and at what values missing points should be replaced.

This laborious procedure is required not only for obtaining a correct solution. It also provides a graphical summary of the key issues and data, which enables others to interpret and assess the decisions that have been made.

What this all means is that transformations from Goldfields to Lo are untrustworthy and will not yield results at an acceptable level of accuracy. The options to get around this problem appear to be either a network adjustment, or the laborious process described above.

Goldfields systems are based on a local plane and therefore do not have sea level corrections or projection enlargement built in. This results in a significant scale difference between Goldfields and Lo. In the test data in this study all Goldfields coordinates had already been converted to approximate Lo coordinates. These factors were therefore already built into the data and so were not an issue.

They could be included as a factor in the distance observation equations by assessing the size of the sea level correction in the area in which calculations are being done. The scale enlargement correction is a function of the Y Lo coordinate and can simply be calculated for each observation. The two corrections could be combined as a simple multiplication factor. Then instead of using a ' l ' in the distance observation equation (4) as explained in Section 6.2.1, a ' l ' multiplied by the relevant factor could be entered giving an equation of the form:

$$(s_{ij} + v_{s_{ij}}) (1 + (Sea\ Level + Projection) + \lambda_n) = \sqrt{(Y_j - Y_i)^2 - (X_j - X_i)^2} \quad (2.1)$$

2.4.6 Local coordinate systems

"Local System" refers to a coordinate system with an arbitrary origin and orientation, which serves as a calculation mechanism for a survey of limited extent, (for example a survey of a single farm). Such systems are common and are usually not related to any other system.

These systems are usually based on a local plane, with no projection or sea-level corrections, which makes them fundamentally incompatible with the projected and corrected Lo system. Differences are usually not significant over the extent of most local systems, but can become significant over an extended area. Standard texts on Survey and/or Map Projections explain the concepts of sea level and projection corrections.

2.4.7 Sides and angles - (no coordinates)

These diagrams reflect no coordinates, but merely record the angles at each polygon apex, or the direction on the Lo coordinate system, and the length of each side. This information can be converted to coordinates if a common point has been coordinated in another survey, or a coordinate value can be obtained from the survey record. (See 2.4.9).

2.4.8 Transnet's compilation plans of unsurveyed and unregistered land

A compilation plan is a medium scale plan showing many different properties on a single sheet. Scales are typically in the vicinity of 1:5000, and errors in digitising from these are of the order of five to ten metres or more. These are often the only means by which the extent of Transnet's unsurveyed and unregistered land is represented. Transnet has ownership rights in land, to which it has not yet taken transfer, by virtue of the expropriation act and a variety of other similar mechanisms.

The origin of the information on plans representing unsurveyed land is often unclear, resulting in very low confidence in their accuracy. It is obviously not possible to create accurate points for unsurveyed land, and this source of information is not under consideration in this thesis.

The data for such polygons will probably be obtained by digitising the compilation sheets. These data will be updated when the property is surveyed.

2.4.9 Survey records

Every cadastral survey performed since 1927 has a formal *Survey Record* (SR) stored in the Surveyor General's offices. In some offices these records date from the previous century. (Zacks J, 1995: Telephone conversation).

These SR's are chronologically referenced and filed sequentially by year (1/1995, 2/1995, ..., 423/1995, etc.). Each record contains a list of points surveyed and/or calculated. These points usually extend beyond just those required to define the polygons (parcels) on the diagrams being created during the survey. The reason for this is that every survey must be tied into existing evidence of other surveys, such as beacons defining the apices of other polygons that are found and surveyed.

Sometimes no new diagrams are created by a survey. The existing beacons are only relocated or replaced. Such surveys are often hard to retrieve from the Surveyor General's filing system because the survey record number is not always recorded on the diagram, as it is with a subdivisional survey.

In the past these surveys may have been carried out using any of the coordinate systems previously described. Current legislation forces all surveys to be linked to Lo. (Regulation 10, Survey Act No 9 of 1927).

2.4.10 WGS84

The Lo system is based on a projection of the Clarke 1880 reference ellipsoid. The reference ellipsoid is a mathematical surface representing the geoid (sea level surface) from which points are projected onto the mapping plane.

The Clarke 1880 ellipsoid has its origin roughly three hundred metres from the gravitational centre of the earth. It was chosen for Southern African mapping because it models the specific distortions in the geoid in Southern Africa very well. In other words - it fits well.

Historically, the datum was carried forward throughout the region by surveying reference or trigonometrical beacons. Subsequent surveys are then linked to these beacons to ensure continuity in the system. Old survey methods did not include the ability to measure long distances accurately. This resulted in scale error in the network that causes distortions in the values of the beacons. In other words the datum is not accurately transferred through the system.

The advent of GPS (Global Positioning System) has changed this scenario dramatically. GPS is a satellite based survey system. Satellites orbit the gravitational centre of the earth. It is therefore sensible to adopt an earth centred reference ellipsoid for measurements made by and to the satellites. The WGS84 is the chosen ellipsoid for this purpose.

Theoretically there is a direct mathematical relationship between coordinates on the Clarke 1880 ellipsoid and the WGS84 ellipsoid.

Satellite based surveys are orders of magnitude more accurate than old methods over long distances. The satellite system provides more precise relative positions (centimetre) than the trigonometrical beacons (decimetre) which they use as a reference. It is necessary to reference to trigonometrical beacons to achieve high absolute precision because there is a built in systematic error in the GPS system that can only be removed by reference to a point of known position. This systematic error drifts unpredictably over time and referencing must accordingly be done on a continual basis.

The distortions in carrying the Lo datum forward throughout South Africa are now significant and must be removed. This is being done using GPS and means that the adoption of WGS84 as a reference ellipsoid is now the logical thing to do.

A complete change in the reference system will take place that will remove the errors. Having surveys connected to trigonometrical beacons, and basing a GIS on the same principle, means that any future change to WGS84 will be easier to adapt to. Not being connected to trigonometrical beacons could mean that captured data are devalued unless there is a planned mechanism for converting from the old to the new reference system.

2.5 CHAPTER SUMMARY

No directly relevant literature was found, and none of the people in charge of the data capture process at a variety of organisations were aware of any documentation on the topic.

There was a general consensus that traversing between fixed points and acceptance of results with misclosures in the 0.10m range were acceptable. In the case study the data sets are not all in municipal areas, and as a result are generally more complex than municipal data sets.

Digitising was rejected as an alternative technique because of the low precision of the method. There are many calculation techniques, such as the Helmert transformation and intersection that may be employed. They generally all suffer from the same shortcomings as far as the creation of a digital cadastral database is concerned. One shortcoming is that only a part of the available cadastral data is used. Others are the failure to take account of the precision of points on which the techniques are based and the failure to calculate or store precision estimators.

Least squares is a technique for solving for the unknowns in an overdetermined solution. In other words when there are more observations than unknowns then there are liable to be inconsistencies. Least squares accommodates these inconsistencies by allowing a residual error to be associated with each observation. The values for the unknowns are then determined so that the sum of the squares of the residuals is a minimum.

Data on diagrams in the former Southern Transvaal, which make up the case study sets, are available in several different survey systems. These are listed with the characteristics of each system most important to the study.

CHAPTER THREE
PRACTICAL CONSIDERATIONS

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3.1 THE GEOMETRICAL BASE OF A GIS IS NOT THE CADASTRE

It is important to remember that the geometric base of a GIS merely contains a graphical/geometric representation of the polygons (Land Parcels) in the cadastre, and is not itself the legal cadastre. It is therefore permissible to use methods in creating the spatial database that would not be permissible under law in the performance of a cadastral survey. An example of such a method is the non-alignment of beacons explained in Section 3.2.

The geometry of the cadastral base of a GIS is based on the coordinates as surveyed and recorded on Land Surveyors' diagrams. As explained in Section 1.3, incompatibilities in the coordinates determined during different surveys for the same point, result in the distortion of most of the polygons in the GIS relative to the polygons as represented on the individual Land Surveyors' diagrams.

This distortion is one reason the geometric part of a cadastral based GIS cannot be taken to represent the legal cadastre. The information associated with and linked to the polygons can contain all the information that makes the system equivalent to any other form of cadastral information storage. The system as a whole can therefore certainly replace the conventional paper-based cadastral information systems. The difference is best illustrated by considering the area as recorded on a particular land surveyor's diagram. This may be different from the area that would be derived from the geometric data in the cadastral base of the GIS. The geometric base is therefore not equivalent to the cadastre.

In a multipurpose GIS the polygon boundaries should be determined relative to topography, and be free of the constraints of the legal cadastre. In spite of the fact that it sounds like a contradiction in terms, the boundaries could perhaps best be described as "cadastral topography".

3.2 ALIGNMENT ISSUES

Having decided to use the latest Lo trigonometrical beacon values as a reference system for the GIS, and to adopt the most recently surveyed diagrams coordinates as probably most correct, (see 2.4.1, 5.1 and 1.13.2 under the sub heading '*LATEST LO VALUE IS MOST CORRECT*'), questions about the alignment of beacons must follow.

Suppose a 1970 metric Lo diagram shows a 2-km long boundary (AB in Figure 3.1) as a straight line. A 1990 subdivisional survey shows two subdivisional beacons (C and D in Figure 3.1) as being on the boundary, but the coordinates indicate that they are 0.20m off the line AB when referenced to the 1970 coordinates. These points may or may not be physically on the line.

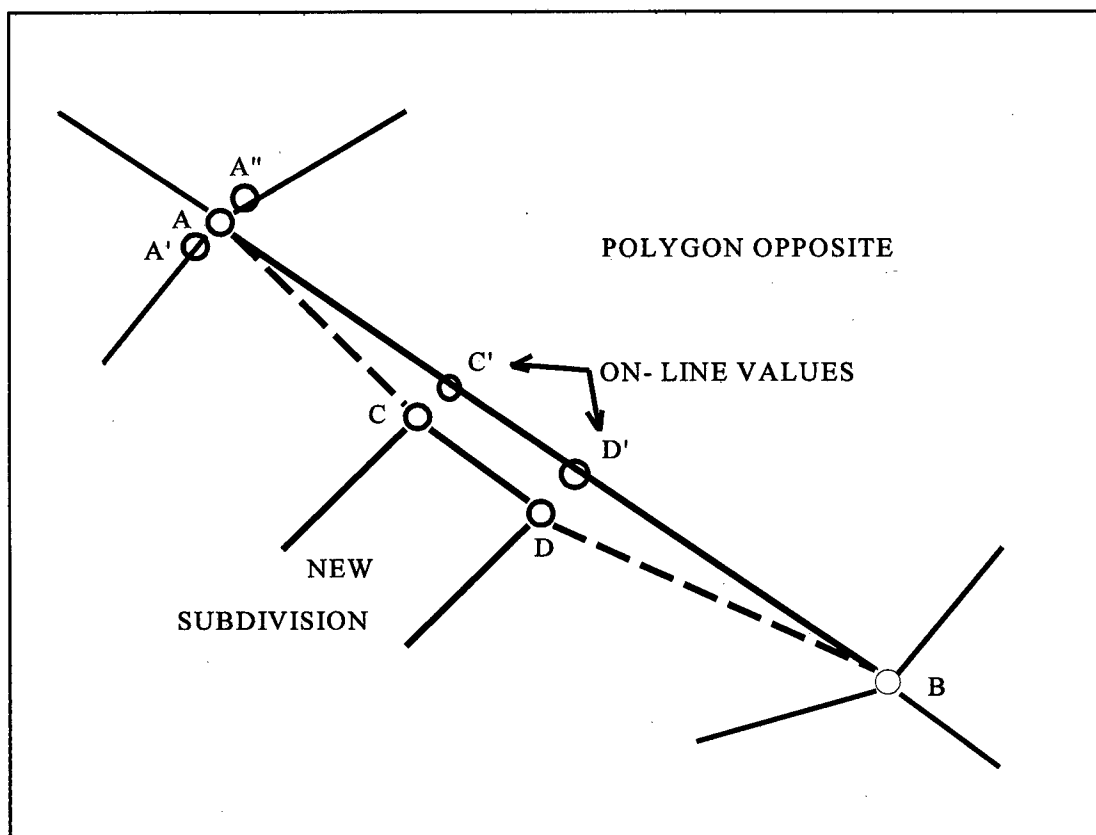


FIGURE 3.1 ALIGNMENT OF SUBDIVISIONAL BEACONS

The question is should one force the new values onto the old boundary to keep the line

straight in the GIS, and the form of the polygon on the opposite side of the boundary simple? Or should the newest most accurate values be used which indicate a bend in the long boundary and are most likely a truer indication of the actual situation on the ground.

The newest values should be used because:

- This is in line with the principle of adopting the latest Lo coordinate.
- This ensures that the system is always up to date with the latest datum, and discrepancies with newer data should occur less often and be smaller.
- The **distortion** introduced to the polygon opposite the subdivision **will be minimal** (invisible at plotting scales), if the survey was done within the legal survey limits defined by the Survey Act. The GIS in the study is designed to deal with such occurrences automatically in the sense that the polygon merely acquires extra apices. It should also be remembered that the geometric base of the GIS is not the cadastre, but merely a representation of it, and that there are therefore no legal ramifications in applying such a principle. (See Section 3.1)
- **It is the most probable representation of reality** because the boundary probably has a bend in it. Land Surveyors have a bias against this type of approach because, in terms of the survey regulations, beacons that fall within a fairly coarse limit off the line are accepted as on line. Aligned values are traditionally calculated for such points. This ignores the fact that the previous surveyor may have performed the same corruption of the point on which the current alignment is being based, and it may have occurred in the opposite direction. Admittedly the size of the corruption may not be very significant, but the surveyors' reason for not introducing the bend is to maintain the geometry of the original straight line, to prevent him having to reject the beacon as unacceptable. There is no reason to perpetuate the illusion that the line is straight for the sake of maintaining geometry in the GIS, especially when the surveyors' approach is based on legal considerations, and not on practical accuracy requirements. The very process of combining data from various sources has led to the distortion in shape and size of many polygons, and no reason exists for not doing the same in this circumstance when the new value is most likely correct.
- **Calculating a value on the line immediately downgrades** the most recent, most

accurate coordinates.

- The **position of features** on the land will be **determined with reference to the Lo systems latest trigonometrical beacon coordinate list**. It is the position of these structures relative to the actual boundaries, as defined by the actual beacons on the ground, which is of interest. This is best done by adopting the latest most recent coordinate.

The method in this thesis performs no checks for the alignment of beacons, because the connection of boundary terminals is performed manually during the checking of the data in the CAD phase, before classification in the GIS. In the situation represented in Figure 3.1 the operator would delete the boundary line segment AB and redraw the boundary as line segments AC, CD and DB. With user-interactive calculation techniques, after making a value-based decision, the operator may have elected to intersect the lines through C and D to the points C' and D' and then delete the points C and D. With data processed through the network adjustment he could immediately apply the method described in the previous paragraph, without needing to investigate the data.

3.3 MAINTENANCE: FUTURE CHANGES IN TRIG VALUES.

There are occasionally minor adjustments in the values of the coordinates of trigonometrical beacons. The entire system will undergo nonlinear changes in the hundreds of metres, with the adoption of the more geocentric WGS84 reference ellipsoid.

Two approaches may be used to ensure compatibility between new and old data:

- Keep to the original reference system and transform all new data back to this datum.
- Transform the relevant portion or, the entire system when there is an adjustment to the value of the trigonometrical beacons.

Adjustments should be made to the latest list of trigonometrical beacons immediately. Adjustments are not made frequently, and the process would therefore not occur often enough to be considered time-consuming. Most adjustments are localised and their limited influence

could be easily dealt with. When an adjustment takes place over a large area, then the entire system could be transformed.

These actions make additional new data readily compatible with what exists. They could be accomplished fairly simply by either the application of a coordinate shift in the most simple case, and an N-common points (Helmert) transformation using the trigonometrical beacons as control points in more complex cases. These techniques could be used equally well for large changes or small local adjustments.

Alternatively all the polygons in a defined area could be reprocessed by a method such as that defined in this thesis, resulting in a complete regeneration of the digital spatial data base for input into the GIS. Links into the attribute data in the GIS are not lost on reclassification with the GIS used by Transnet in this study, because each polygon is classified with its key field identifier as a text string located within the polygon. Boundary compatibility with adjacent polygons could be achieved by causing the adjacent polygons to be linked to trigonometrical beacons. This could be done by the addition of trigonometrical beacons to their data if they were not so linked. The effect would be that all the adjacent polygon points would be held fixed in an adjustment under the mathematical model defined in this thesis.

A comprehensive history of actions taken in this regard should be maintained, and a specific policy formulated to ensure consistent decision-making, should be established by any organisation affected by the issue.

3.4 DISTINGUISHING BETWEEN DIFFERENT POINTS THAT ARE CLOSE TOGETHER, AND DIFFERENT VALUES FOR THE SAME POINT.

An intuitive method for deciding which points of differing value are representations of the same point is to examine all points within some radius of each other.

The problem with this approach is that there are situations in which points from different polygons lie physically close together, and therefore have values that may place them within

the specified radius of each other, when they represent different points.

Three situations in which this is the case are:

- Indicator beacons. These are physical points placed to reference a polygon's apices that cannot be physically marked. These points do not define polygon apices, but should not for this reason be excluded from a point determination routine. They provide essential connectivity between the polygons in the adjustment, as explained in Section 8.2.3.
- Polygons that have short sides. These can be identified by calculating the length from every point to all other points of a polygon, and reporting on those that are within a specified distance. These points can be investigated manually before running any automatic process.
- Polygons with apices that are close together but not coincident. In Figure 3.2 two polygons with nearby points 1C and 2A on the same boundary are shown. Due to the way data are structured on South African survey diagrams, neither polygon carries information about the other's nearby apex.

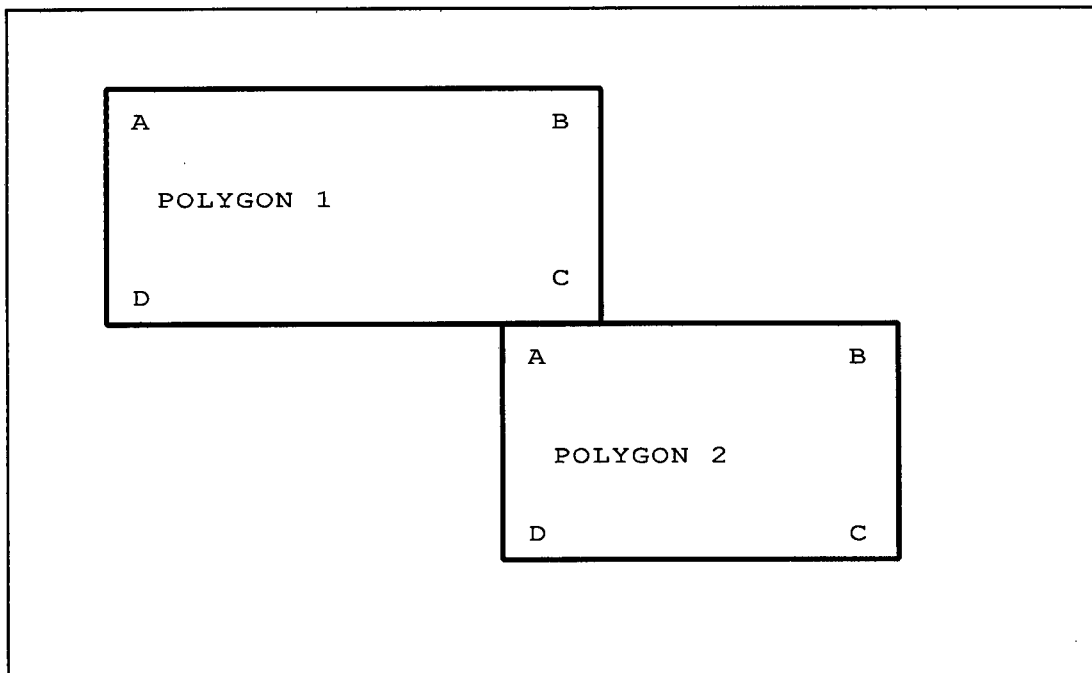


FIGURE 3.2: TWO NEARBY POINTS IN DIFFERENT POLYGONS

No simple efficient method exists for detecting such points directly from a database of points, before the generation of the GIS topology. Detecting the short side of a polygon does not work as the points lie in different polygons and will therefore not be identified. The danger is that these points will be confused as being the same point.

The automatic process defined in this thesis has no special mechanism for dealing with the particular variation of the problem where two points closer than 0.30m are not the same point. It does, however, report on all points within two metres but more than 0.30m apart.

If non-coincident points are dealt with as being the same point, then it should be apparent from the residuals of one or all of the observations to the point that there is a problem. If the difference is too small to detect, and the error is within the precision limits defined for the system, then there is not a significant practical problem. The GIS should, however, be structured flexibly enough to enable the correction of such an error should it be detected at a later stage.

3.5 CHAPTER SUMMARY

The digital spatial cadastral database underlying a GIS contains representations of the component polygons in the cadastre, but is not itself the cadastre. It is therefore permissible to adopt procedures in the creation of the spatial database that suit the purposes of the GIS, but would not be permissible in law in the cadastre itself. Alignment of beacons is such an issue.

Beacons supposedly lying on-line in the cadastral system, are not tested for alignment, nor aligned by the method. It is assumed that the coordinate determined by the adjustment is the most likely value for the actual point relative to trigonometrical beacons. It is the relationship between the actual point and other topographical features that is important in a multipurpose GIS, such as the one in the study.

There are practical problems in differentiating between different points that are close together, and the same point that has differing values from different polygons. The approach adopted in

setting up the required referencing for the least squares program is to reference points as the same point when coordinates fall within 0.30 metres of each other, and to list others within one or two metres for manual investigation. The tolerances for referencing and listing are user definable. If non-coincident points are dealt with as being the same point, then it should be apparent from the residuals of one or all of the observations to the point that there is a problem. If the difference is too small to detect, and the error is within the precision limits defined for the system, then there is not a significant practical problem.

CHAPTER FOUR
PREPARING DATA

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4.1 INTRODUCTION

Transnet's Land Surveying Services department has created a data base known as the ECL (Electronic Coordinate List) for all the points to be used in their digital cadastral database. The ECL combines both point and polygon data for all polygons in a single file. Part of this file is included as an example in Appendix B. The data are extracted from this file by specifying a set of serial numbers referencing the various polygons wanted for the specific exercise. The form and purpose of the most important fields of the extracted data are shown in Table 4.1 and explained in Section 4.2. The file has a DOS extension ".ECL".

The database in which this large single file is stored is a hierarchical database. There are many duplications in this database, and errors in some entries in duplicated fields have resulted in duplications of polygons and points. The program BIGECL contains some routines with the sole purpose of detecting these errors, and reporting them. These errors then have to be corrected in the parent database, the subset re-extracted and the program BIGECL rerun to ensure the correctness of the data.

This repetition through error represents a massive duplication of effort. The failure to manage the data entry phase of the project because of the limitations of the ECL program, has meant that some polygons have been entered two or three times. Although this is not a research problem in terms of this study's objectives, data management is an area that needs to be addressed in any production application of this method, or else the number of wasted man hours can become unacceptable.

It is recommended that the ECL be translated to a relational database to facilitate the management of the process, and the interaction between it and the windows based software used for both this thesis and the GIS.

The prefixes CMY and SNX in the polygon identifiers are used to differentiate between routes, and together with the polygon number form a unique serial number for each polygon.

4.2 TRANSNET'S DATA CAPTURE PROCESS

The ECL database has the following fields:

- The name of the point as recorded on the land surveyor's diagram. (NAME in Table 4.1)
- Y and X coordinates on the original survey system. (Y_IN and X_IN in Table 4.1) The coordinates as they appear on the diagram, or as they are calculated.
- Y and X coordinates on the Lo system. (Y_COORD and X_COORD in Table 4.1) These represent the values on the Lo system as they have been transformed from the original. For example conversion from cape feet to metres, or Goldfields transformations. These coordinates have been used as the provisional estimate of the unknowns in the least squares program.
- Y and X coordinates GIS. (Y_GIS and X_GIS in Table 4.1) These represent the value determined to represent the point in the GIS. All points representing the same point will have the same value in this field to ensure that no sliver polygons are created. The selection of a value to fill this field is the central theme of this thesis.
- The serial number of the polygon (SER_NO in Table 4.1). This is unique and distinguishes it from every other polygon.

TABLE 4.1 ECL DATA BASE MAIN FIELDS.

NAME	Y_IN	X_IN	Y_COORD	X_COORD	Y_GIS	X_GIS	SER_NO

A variety of other fields not relevant to the discussion at this stage have been created to identify the source of the data. The NAME and SER_NO combination uniquely identify each point.

This data entry has been done in the following way:

- Scanning and conversion to text of all standard font data on Lo diagrams (48 000 coordinates in the Johannesburg Office)
- Typing of all nonstandard font data on Lo diagrams
- Scanning and conversion to text of all standard font data on non-Lo diagrams

- Typing of all nonstandard font non-Lo diagrams
- Calculation of remaining points using sides and angles from points of known Lo value
- Graphical manipulation using Helmert transformations, adjusted traverses and polars to fit data between existing points.

4.3 PROGRAM BIGECL - PURPOSE

As part of this thesis the author wrote the program BIGECL to read the extracted subset of the original ECL file. (See BIGECL.PAS in Appendix L for a listing). First it determines the number of polygons as CPly and points as CPnt, and then creates the arrays PolyArray and PointArray in memory to hold the data. It then splits, prints (see Appendix E.1) and stores the information in separate polygon and point files with the DOS file extensions .PLY and .PNT.

It simultaneously calculates the centroid (centre of gravity or mean coordinate) for each polygon. It then calculates the distance to the furthest point of the polygon, and stores it as the radius. This is the radius of influence of the polygon beyond which intersection with another polygon is not possible. This information is used in the program BIGFIXLO to reduce the amount of processing to only those polygons that could possibly intersect, when checking for duplication of points to establish the referencing. It is also used to calculate the effect of scale and orientation corrections on the polygon. (See 6.4.6).

The centroid is also used to check for polygon duplication because of farm, portion or serial number errors. If polygons have the same centroid, or a centroid within two metres of another polygon then this information is printed, enabling investigation and correction. 30% of the polygons in the test data set were duplicated because of farm number errors.

The program also sorts the polygon records in a descending temporal order from the highest dated survey record year and survey record number to the lowest. This is done because of the assumption that the most recent survey is more likely to be more accurate. When subsequent polygons repeat a point from a prior polygon, the point is referenced to the most recent point by the program BIGFIXLO.

The program then checks for duplications in polygons and reports on these to enable correction.

The duplicates checked for are:

- Farm and Portion name combination
- Serial number
- Centroid (exact match)
- Possible centroid match (within two metres).

Trigonometrical beacons recorded on the diagrams are then checked for existence, and gross error, against the trigonometrical master file holding the latest data.

Finally the program de-allocates the memory reserved for the arrays PolyArray and PointArray.

An explanation of the procedures in the program is included as Appendix C1.

The program output in the "Polygon Detail List" (see Appendix E.1) consists of the sequential number of the polygon (as determined by the program) followed by the fields as listed (with the purpose as explained in Section 4.3):

Farm, Ptn, Units, SRYear, SRNum, SERNum, RecStt, RecEnd, System, CntrY, CntrX, Radius.

The "Points Detail List" (see Appendix E.1 and E.2) has the sequential number of the point (as determined by the program) followed by the fields as listed (with the purpose as explained in Section 4.3):

PntName, YSrce, Xsrce, YProv, XProv, PntTyp, PntNat.

4.4 BIGECL DATA FIELDS

FIELD NAME	PURPOSE
------------	---------

Point File (.PNT) fields.

(Extracted from the original ECL file)

PntName	Holds the point name
YSrce	Holds the Y source coordinate (Y_IN in Table 4.1)
XSrce	Holds the X source coordinate (X_IN in Table 4.1)
YProv	Holds the Y provisional Lo coordinate (Y_Coord in Table 4.1)
XProv	Holds the X provisional Lo coordinate (Y_Coord in Table 4.1)
YGis	Holds the Y GIS coordinate as determined manually
XGis	Holds the X GIS coordinate as determined manually
PntTyp	Distinguishes between points associated with the figure defining the polygon (POLY) and other points such as trigonometrical beacons or connections (POINT)
PntNat	This further details PntTyp POINT into the specific point type (TRIG for trigonometrical beacons, KON for connections)
Irreg	Indicates an irregular boundary such as a curve or river

(Fields added by the program)

YGisNW	Holds the Y GIS coordinate as determined by the network adjustment
XGisNW	Holds the X GIS coordinate as determined by the network adjustment
dyGIS	Holds the standard deviation of YGisNW
dxGIS	Holds the standard deviation of XGisNW
Stat	Indicates whether the point is, F = Fixed, P = Provisional or R = Referenced for normal points, and Y = Used or N = Not for trigonometrical beacons
Ref	For referenced points holds the record number of the point it is referenced to

*Polygon file (.PLY) fields**(Extracted from the original ECL file)*

Farm	The farm or township number
Ptn	The erf or portion number
Units	The units of the original survey with, for example, c = cape feet, m = metres
SRYear	The year of the survey record number
SRNum	The sequential number of the survey record
SERNum	Transnet's database serial number for the polygon
System	A code for the survey system

(Fields added by the program)

RecStt	The record number of the first point of the polygon in the points file
RecEnd	The record number of the last point of the polygon in the points file
CntrY	The Y Lo coordinate of the centroid (Centre of gravity) of the polygon
CntrX	The X Lo coordinate of the centroid (Centre of gravity) of the polygon
Radius	The distance from the centroid to the furthest point of the polygon

4.5 CHAPTER SUMMARY

Transnet's ECL (Electronic Coordinate List) is a single hierarchical database file. This file is fraught with errors and duplications. The program BIGECL was written to separate the polygon and point specific fields, and to detect a variety of errors. It simultaneously determines the number of polygons, the number of points per polygon, and the centroid and radius of influence of each polygon.

The polygons are sorted sequentially in a descending temporal order by the survey record year and number. The program creates and stores the information in two files, the polygon information file has a '.PLY' extension and the point file a '.PNT' extension. The polygon file references the points by sequential line number in the points file.

CHAPTER FIVE
FIXING AND REFERENCING POINTS

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5.1 APPLYING TRIGONOMETRICAL BEACON SHIFT

5.1.1 Principle

Lo based surveys are carried out with reference to the published coordinate values for trigonometrical beacons. These coordinate values are amended from time to time as better observations are obtained, or errors are detected and eliminated.

The coordinate values on the diagrams are, however, not amended. This amounts to a local change in datum, (as explained in Section 2.4.2), and this must be allowed for in reconciling pre- and post-adjustment surveys. This adjustment is facilitated by the recording of the coordinates of two of the trigonometrical beacons used in the survey on each diagram.

In instances where these coordinates have not been recorded, it is not possible to allow for the effective datum change. The survey is then treated as a non-Lo survey, and the data used in the network adjustment program.

The change in the trigonometrical beacon coordinates may be any combination of translation (shift), rotation and scale. The effects of rotation and scale are usually negligible. The use of a simple mean shift was therefore considered sufficiently precise to not detract from the aims of the thesis. (See 1.13.3 *REFERENCE POINT ADJUSTMENT*).

5.1.2 Practical

The mean shift, determined between the trigonometrical beacons recorded on the diagram and the latest published list, is applied to all the points in a polygon. The highest dated, highest numbered Lo polygon's points are referenced to by any subsequent duplications of the same points.

In broad outline the procedure is:

- 1 Work through the polygons from highest dated, highest numbered survey record down

to the lowest.

- 2 Check if a polygon is surveyed on Lo.
- 3 Calculate trigonometrical shift if trigonometrical beacons are recorded on the diagram.
- 4 Apply the mean shift to the polygon apex coordinates.

5.2 REFERENCING POINTS

5.2.1 Principle

In a network adjustment each point recognised as unique will have associated unknowns to be resolved. If two polygons have differing coordinate values for the same point, and those points are not recognised as representing the same point, then two points will be determined by the network adjustment. This is exactly the situation that this method is attempting to eliminate and resolve with a high degree of automation. (See Section 3.4).

Those points that are found to lie within 0.30 metres of each other are assumed to be the same point, and referenced accordingly. Points which fall within two metres of each other are listed for investigation. Polygons with sides less than one metre in length are also listed for investigation. The user may manipulate the point referencing to ensure that it is correct. All the tolerances used in the tests may be changed in the program if problems are encountered. This helps to resolve problems.

Only one serious referencing problem was encountered, which was the result of an error on a diagram. This was corrected by means of a data reconstruction. (See Section 8.3.3). Most other referencing problems were caused by data entry errors, which were easily corrected. This phase of the programming is therefore considered successful.

5.2.2 Practical

To facilitate this process in the study, the preliminary coordinates on the Lo system, as determined and stored in the ECL, were used to compare point positions. This meant that all

the data had coordinates on the Lo system.

Many of the data were not originally coordinated on the Lo system. A method for quickly and easily calculating provisional Lo coordinate values from source data, while simultaneously sorting out the referencing would be extremely useful. The development of such a method is proposed in the recommendations.

The processing steps are:

- 1 Mark all points in the first polygon that is referenced to trigonometrical beacons as fixed.
- 2 Work through the polygons from the most recent to the oldest Lo polygons referenced to trigonometrical beacons.
- 3 Compare each polygon's points with those of the already processed polygons. Work from the oldest to the most recent polygon, and record the point number of any point referenced to in the referencing field of the point data array. This ensures that the most recent point is the one held fixed and referenced to by all other polygons.
- 4 Next work through all the other polygons in the same manner.
- 5 If a point does not exist in a previously processed polygon, fix its value to the current shifted coordinate if it is a referenced Lo polygon. Mark it as provisional if it is not a Lo referenced polygon. If it does exist in a previous polygon, reference it to the previous polygon point.

A listing of the program BIGFIXLO is included as Appendix M. An explanation of the procedures in the program is presented in Appendix C.2, and an example of the output is included as Appendix E.3 - E.5. The printout in Appendix E.3 - E.5 lists the sequential number of the point followed by the fields (with purposes as explained in section 4.3):

PntName, YSrce, XSrce, YProv, XProv, Stat, Ref, dYGIS, dXGIS, YGISNew, XGISNew, Stat, Ref.

5.3 CHAPTER SUMMARY

All Lo polygons connected (referenced) to trigonometrical beacons are adjusted for changes in the trigonometrical beacon coordinates and marked as fixed points. The points are compared to all other polygons already processed, and should a point be found to represent a previously encountered point, it is referenced to that previous point and assumes its coordinates.

All non-referenced Lo polygons and all non-Lo polygons are processed for point duplication and referencing in the same manner. This referencing is critical to the operation of the least squares program, as it prevents two points being determined for what is in fact a single point, by correctly setting up the provisional points and determining how the pseudo observations are formed. The creation of the pseudo observations in line with the defined mathematical model is a foundation of the method. (See 1.7).

6.1 INTRODUCTION

In the mathematical model in this thesis, data are derived from the sides, angles, directions or coordinates on a diagram, and are treated as observations between the points representing the apices of the polygons. These are not true observations and are referred to as pseudo observations.

Initially a scale error and orientation correction were allowed per polygon. This was later altered and a single scale and orientation correction have been allowed per survey record. (See 8.3).

The mathematical model uses the principle of keeping L_0 points fixed, after they have been adjusted for Trigonometrical shifts. See Chapter 5.

6.2 OBSERVATION EQUATIONS

The Figure 6.1 is a general schematic representation of the situation being resolved by the least squares solution. Data are derived from the diagram representing the polygon. The letters A, B, \dots, N represent N polygons with varying numbers of sides. The apices of the polygons are labelled a, b, \dots, n , and the quasi observations labelled $1, 2, \dots, n$. Both n and N represent some unspecified number, and the observation ' t ' represents both the direction observation ' t_i ' and the distance observation ' s_i '.

In the explanation below, Y and X represent the final coordinates of a point. (Y) and (X) represent the approximate or initial estimates of the point coordinates (provisional coordinates).

The final solution including the unknowns dY and dX is then expressed as:

$$Y = (Y) + dY \quad (1)$$

$$X = (X) + dX \quad (2)$$

CHAPTER SIX

A LEAST SQUARES PROGRAM FOR POINT COORDINATE DETERMINATION

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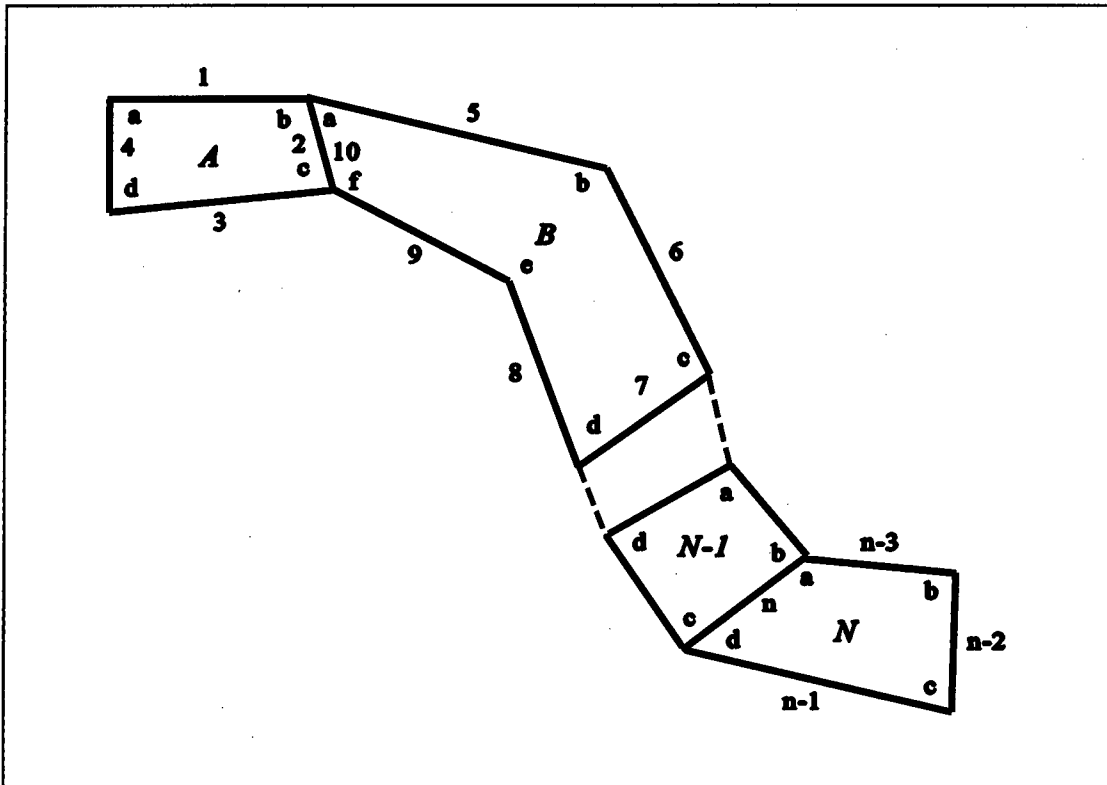


FIGURE 6.1: GENERAL SCHEMATIC REPRESENTATION

6.2.1 Definition of observation equations

The observations t and s are written in terms of the unknowns Y , X , θ and λ , where i represents the i th point, j the next point and n the n th polygon. v represents the residual or observational error, θ represents the orientation correction for a polygon and λ the scale correction for a polygon in the first version of the mathematical model. This was amended to allow one scale error and orientation correction per survey record for reasons as explained in Sections 8.2.3 and 8.3. In the amended model n refers to the n th survey record.

The observation equations in the case of directions can be written generally as:

$$t_{ij} + v_{t_{ij}} + \theta_n = A \tan \left(\frac{Y_j - Y_i}{X_j - X_i} \right) \quad (3)$$

The observation equations in the case of distances can be written generally as:

$$(s_{ij} + v_{s_{ij}}) (1 + \lambda_n) = \sqrt{(Y_j - Y_i)^2 + (X_j - X_i)^2} \quad (4)$$

A scale factor allows for the case that the error in the distance is not a purely random error in the observation, but that it also has a systematic scale component. This would be especially significant in coordinate systems that do not allow for a projection enlargement and sea level correction as does the Lo coordinate system chosen for the unknowns. Alternatively, these corrections could be built into the observation equations as shown in equation (2.1) in Section 2.4.5.

Linearising equation (3) and subtracting various terms from both sides results in equation (5).

$$v_{t_{ij}} = + a_{ij} dX_i - b_{ij} dY_i - a_{ij} dX_j + b_{ij} dY_j - d\theta_n + F_0 - \theta_n - t_{ij} \quad (5)$$

In which

$$a_{ij} = \frac{\Delta Y_{ij}}{s_{ij}^2} = \frac{Y_j - Y_i}{(Y_j - Y_i)^2 + (X_j - X_i)^2} \quad \text{and} \quad b_{ij} = \frac{\Delta X_{ij}}{s_{ij}^2}$$

$$\text{and} \quad F_0 = \text{Atan} \left(\frac{(Y_j - Y_i)}{(X_j - X_i)} \right)$$

Similarly the distance equation (4) may be linearised giving equation (6).

$$v_{s_{ij}} = - c_{ij} dX_i - d_{ij} dY_i + c_{ij} dX_j + d_{ij} dY_j - d\lambda_n + G_0 - s_{ij}\lambda_n - s_{ij} \quad (6)$$

In which

$$c_{ij} = \frac{\Delta X_{ij}}{s_{ij}} = \frac{X_j - X_i}{\sqrt{(Y_j - Y_i)^2 + (X_j - X_i)^2}} \quad \text{and} \quad d_{ij} = \frac{\Delta Y_{ij}}{s_{ij}}$$

$$\text{and} \quad G_0 = \sqrt{((Y_j - Y_i))^2 + ((X_j - X_i))^2}$$

6.2.2 Units

The units of the distance observations and coordinates are usually metres. Observation residuals and corrections to unknown coordinates are also generally expressed in metres (m). These conventions have been followed in this thesis.

The units of direction observations are generally degrees. The units of the observation residuals are usually expressed in seconds. In the observation equation it is usual to convert all direction related values to seconds. These conventions have been followed in this thesis.

The distance observation equation has the form of equation (6) which is an equation in m (metres), and is consistent in its units. The direction equation (5) is an equation in " (seconds), and is inconsistent in that the elements of the dX and dY are radians. These are usually multiplied by $\rho_0 = \frac{180 * 3600}{\pi}$ to make the equation consistent in seconds. Equation (5) then becomes:

$$v_{t_{ij}} = + \rho_0 a_{ij} dX_i - \rho_0 b_{ij} dY_i - \rho_0 a_{ij} dX_j + \rho_0 b_{ij} dY_j - d\theta_n + F_0 - \theta_n - t_{ij} \quad (7)$$

6.2.3 Weights - conventional survey network method

The weight (w_i) of an observation is a relative measure of its influence in the solution. The weight is defined as the variance of the observation of unit weight (σ_0^2) divided by the variance of the observation (σ_i^2):

$$w_i = \frac{\sigma_0^2}{\sigma_i^2} \quad (8)$$

Usually the weight of the direction observation is set equal to 1. Direction observations in survey networks using single second theodolites typically have a standard deviation of 3"

(Sokkisha, undated: 92). This is of course instrument and observation procedure dependant, and may vary from a fraction of a second to 20" or more. Substituting a standard deviation of 3" into equation (8) gives $\sigma_0 = 3$, if unitary weight is assigned to the direction observations.

Distance observations with modern EDM's (combining the fixed and the parts per million (ppm) components) typically have a standard deviation of approximately 5mm. This is once again instrument and observation procedure specific. Substituting this into equation (8) gives a weight for the distance observations of 360 000 if σ_0 is taken as 3 and if the length of the observation is assumed not to affect the weight.

There is generally no correlation between observations in a standard network, and the weight matrix is therefore set up as a diagonal matrix with appropriate weights on the main diagonal, and no off diagonal terms.

6.2.4 Units and Weights - this method

In the method proposed in this thesis both the distance and direction observations are derived from the same set of coordinates. This means first that they are correlated and second that they are of the same precision and must be given equivalent weight. This is achieved by using unitary weight for EDM era distances, equation (12) for pre-EDM era distances and equation (11) for directions.

The standard deviations of both the direction and distance observations are derived from the standard deviation of the coordinates from which the observations are derived, and not from actual observations. There are several possible standard deviations for the coordinates on the source diagrams. These are highly variant in practice and may vary from 0.01m to 0.50m. It would be a practically impossible task to evaluate the manner and the associated precision of the determination of each point.

Some assumption is however necessary to create the weight model. The assumptions below are based on the author's thirteen years of experience of surveys typical of those in the case study,

and the knowledge that triangulation was used to control many surveys prior to the advent of the EDM. Triangulation has an inherently lower precision than EDM-based traverses as a method of establishing control over the long distances covered by most of the surveys typical of the case study.

The probable standard deviation of a coordinate derived from survey in a rural area using a tape for distance measurement is 0.10m to 0.20m. The probable standard deviation for a coordinate in a rural area using an EDM for distance measurement is 0.03m to 0.10m.

The probable standard deviation for a coordinate derived from survey in a township using a tape is 0.05m to 0.10m, while the use of an EDM would probably yield standard deviations of 0.03m to 0.06m.

The distinction between rural and township surveys adds an unnecessary and hard-to-categorise difference, which will probably contribute little to the result. The distinction between tape and EDM is significant, but none-the-less difficult to categorise in the transition phase between the advent of the EDM and its general use by land surveyors. Surveys before and during the early part of this century often exhibit far lower levels of precision than has been assumed. Standard deviations of more than one metre are not uncommon.

For the sake of ease of application in the programming, surveys prior to 1985 will be presumed to have been performed with a tape, and those from 1985 on, by EDM. This assumption is unlikely to affect the results to such an extent that it will invalidate them in terms of the aims of the thesis. The weight model needs further investigation but will not be examined further in this study.

The standard deviation in the angle in seconds can be directly expressed in terms of distance by the following equation. See Section 6.2.5.

$$v_n = \left(\frac{v_m}{s} \right) \frac{180 * 3600}{\pi} \quad (9)$$

Conversely the angle expressed in metres relative to its base may be expressed directly in seconds by the equation:

$$v_m = \frac{v_n * s * \pi}{180 * 3600} = \frac{v_n * s}{\rho_0} \quad (10)$$

This relationship is used in converting a weight from its distance equivalent to a value in seconds.

If we assume a standard deviation for a distance of 0.05m, and assign this observation unit weight, then $\sigma_0^2 = 0.0025$. Converting the distance expression of the standard deviation to seconds to achieve equivalent weighting, the weight for a direction observation expressed in seconds is given by:

$$w_n = \frac{\sigma_0^2}{\left(\frac{0.05 * \rho_0}{s}\right)^2} = \frac{\sigma_0^2}{0.05^2} * \frac{s^2}{\rho_0^2} = \left(\frac{s}{\rho_0}\right)^2 \quad (11)$$

$$\text{remembering that } \frac{\sigma_0^2}{0.05^2} = 1$$

The formula for determining the weight of a distance is a function of the standard deviation for EDM era surveys. These observations are assigned unitary weight. Assuming a standard deviation of 0.05m for these observations and 0.15m for pre-EDM surveys gives a weight for pre-EDM surveys of

$$w_m = \frac{\sigma_0^2}{0.15^2} \quad (12)$$

The standard form of the observation equations has been used. Weights as expressed in equation (11) for directions, unitary weight for EDM era surveys and equation (12) for pre-EDM surveys have been used.

6.2.5 Residuals

In contrast to normal survey networks where one observation (albeit reduced from multiple observations) per distance and direction is usual, this method uses data in which multiple conflicting observations for the same direction and distance exist. The resolution of these specific types of inconsistencies is what the method seeks to partially automate.

Examination of the residuals of all observations is necessary, because an unusually large residual may indicate gross error in data capture, or the incorrect identification of points representing the same point, or a fault in the weight model because of incorrect assumptions relating to the precision of a particular category of data.

The interpretation of a direction residual expressed in seconds is totally dependent on the length of the base to which it relates, and it would be easier to assess these residuals if they were expressed in metres. This is easily done by the radian link between angles and distance as illustrated in Figure 6.2 below.

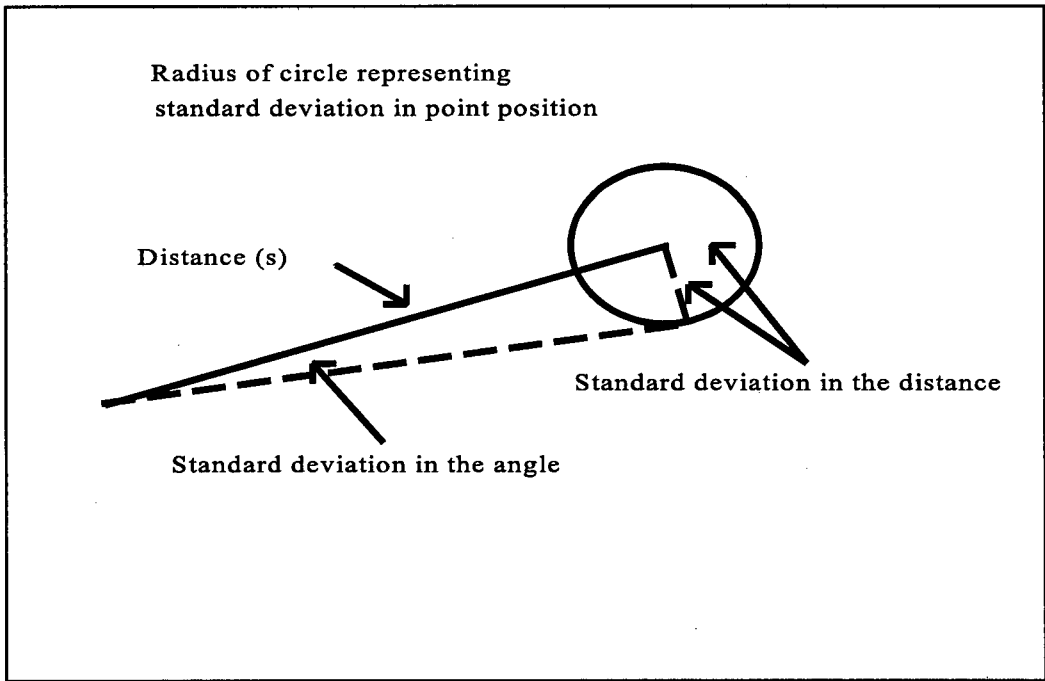


FIGURE 6.2: RELATIONSHIP BETWEEN DIRECTION AND DISTANCE STANDARD DEVIATION.

The direction residuals have all been expressed in metres and also seconds in the printouts of results to make them easier to assess.

6.2.6 Degrees of freedom

Using the first mathematical model any n sided polygon will have $2n$ observations (direction and length of each side) and $2n$ (coordinates) + $2 (\theta, \lambda)$ unknowns. This is obviously insoluble. In m polygons, known values for the f start and end points, and the existence of c points common to other polygons, eliminates two unknowns for each such known or common point. The degrees of freedom for the network under the first mathematical model may therefore be defined as:

$$\text{Degrees of freedom} = \left(2 \sum_{i=1}^m n \right) - \left(\left(2 \sum_{i=1}^m n \right) + 2m - (2f + 2c) \right) \quad (13)$$

This simplifies to, $\text{Degrees of freedom} = 2(f + c - m)$. In other words, as long as there are more known and common points than polygons a solution should be possible. This of course ignores geometrical problems in specific polygons as explained in Section 8.1.3, and does not guarantee a solution. A test for this condition has been built into the program BIGNETSR, prior to attempting the matrix inversion.

Under the second mathematical model where there are r survey records, the degrees of freedom changes to, $\text{Degrees of freedom} = 2(f + c - r)$.

The known points in the case study are those Lo points that have been referenced to trigonometrical beacons, and adjusted for any changes in the reference system. When a point is held fixed there are no unknowns to solve for, and the point is therefore not reflected in the vector of unknowns. It may, however, have an influence because of the calculation of observations to or from it.

6.3 FORMING MATRICES AND VECTORS

The entire group of observation equations forms a group of m simultaneous equations in n unknowns. This can be written in the matrix form $v = Ax + f$. Where v is the vector of residuals or corrections to the observations. A is the matrix of coefficients of the vector of unknowns x , and f is a vector of constants. (See 2.3).

In this adjustment there are, in fact, no genuine observations. The observations are pseudo observations derived by calculation (Join) between the coordinates that appear on the diagrams, or which have been calculated as the first approximation to the final Lo value.

The coefficients of the A matrix (defined in terms of the approximations to the unknowns) will hence be calculated in terms of the accepted provisional coordinates for a point. The accepted provisional coordinates will be from one of the diagrams on which the point exists. The provisional coordinate will be the same as that used in calculating the quasi observations to and from the point, in the case of the observations for the diagram from which the provisional point was selected.

Mechanisms do exist for eliminating the orientation corrections from the A matrix to reduce its dimension. This elimination will not be considered unless computer memory space becomes a serious problem. This is not foreseen as a problem because of the unit BIGMAT, which makes possible the creation of extremely large data arrays. An array of 1500 by 1500 (2 250 000 elements) was created, data read in and read out again, in 24 seconds in 13 Mb of memory. (See 7.2).

6.3.1 Forming the A matrix

The first portion of the A matrix reflecting the first ten observation equations is reflected in Table 6.1 together with the relevant parts of the v , x and f vectors, for the layout as shown in Figure 6.1. The table reflects the system of using a scale and orientation correction per polygon, which was later amended to use a single orientation and scale correction per survey record as

explained in Sections 8.2.3 and 8.3.

The names by which the unknowns have been labelled in Table 6.1 are those of the first polygon in which the point exists. Duplications of the point in subsequent polygons are not unknowns, but merely the source of observation calculations. For example, point "Ba" does not appear as an unknown because point "Ab" represents the unknown in this instance.

The least squares solution requires accurate recording of which nominal points in which polygons refer to the same physical point. If this is done at the data entry stage, then little extra effort is required to prepare the data for processing in a least squares adjustment program.

This is not so for the current data sets. It is therefore critical that some mechanism for identifying and recording which nominal points are duplicates of the same unknown be identified. This has been done by the program BIGFIXLO and the information has been recorded in a way that can be used by the adjustment program. (See 5.2).

6.3.2 Forming the f Vector

In the case study the provisional coordinates are accurately calculated and the pseudo observations are calculated between these provisional coordinates. This means that the initial estimates for the orientation corrections, θ_n , will be zero. If actual direction observations in a system such as Goldfields were used, instead of a join between the provisional coordinates as has been used in the case study, then the provisional orientation correction would be approximately ninety degrees. This is because the Goldfields systems are swung by approximately ninety degrees from the Lo system. This level of refinement is only considered worthwhile in a production version of the program because it makes no practical difference to the results in terms of the aims of the study.

The f vector contains the terms $f_0 - t_{ij} - \theta_n$ for directions and $g_0 - s_{ij} - s_{ij}\lambda_n$ for distances. In the cases where the provisional coordinates and the coordinates for calculating the quasi observation are the same then $f_0 = t_{ij}$ and $g_0 = s_{ij}$. Differencing these terms leaves zero, and the

provisional values of θ_n and λ_n will also contribute zero. These terms have been calculated and differenced in the normal manner to ensure the generality of the program.

The program BIGNETSR (see Appendix C.2 for an explanation of the procedures and Appendix N for a complete listing) performs all of the calculations and processing relating to the network adjustment. It is usual in a network adjustment to iterate the solution, until some predefined precision is achieved, or shown to be unachievable.

The adjusted coordinates are then used as the provisional coordinates in the second and subsequent iterations. These are usually more precise than the first approximations for the provisional coordinates, and result in smaller corrections being determined in each subsequent iteration. The corrections are the unknowns being solved for, and the smaller they are, the more "precise" the results. This is because of dropping second-order terms in the linearisation of the observation equations, which are presumed to be insignificant.

No iteration has been used in the test version of the program, and will only be added if the program is converted to an operational version. The reasons for this are:

- The provisional coordinates were accurately calculated, and the corrections were therefore known to be small (less than 0.70 metres).
- The observations have been calculated from the provisional coordinates. If new observations were calculated from adjusted coordinates then the geometry of the original polygons would be lost.
- The coordinates of various polygons were manipulated to simulate a variety of typical data systems. The mathematical model's ability to correct these manipulations in a first pass was a good indication of its robustness.
- In instances where problems arose, these could have been "lost" or disguised by iteration.
- No preset level of precision was defined towards which to iterate because the precision of the input data was not well known. This meant that the expected precision of the result was also not known. This is in contrast to most conventional survey networks. This may become possible after more investigation into the weight model.

TABLE 6.1: v VECTOR, A MATRIX, x AND f VECTORS

v	A															...	theta		...	lambd a		...	x	f	
t1	a	-b	-a	b													-1						dXAa	tAaAb - tAaAb	
s1	-c	-d	c	d																s1a1b			dYAa	sAaAb - sAaAb	
t2			a	-b	-a	b											-1						dXAb	tAbAc - tAbAc	
s2			-c	-d	c	d														s1b1c			dYAb	sAbAc - sAbAc	
t3					a	-b	-a	b									-1						dXAc	tAcAd - tAcAd	
s3					-c	-d	c	d												s1c1d			dYAc	sAcAd - sAcAd	
t4	-a	b					a	-b									-1						dXAd	tAdAa - tAdAa	
s4	c	d					-c	-d												s1d1a			dYAd	sAdAa - sAdAa	
t5			a	-b					-a	b												-1		dXBb	tAbBb - tBaBb
s5			-c	-d					c	d													s1b2b	dYBb	sAbBb - sBaBb
t7											a	-b	-a	b								-1		dXBd	tBcBd - tBcBd
s7											-c	-d	c	d									s2c2d	dYBd	sBcBd - sBcBd
t8													a	-b	-a	b						-1		dXBc	tBdBc - tBdBc
s8													-c	-d	c	d							s2d2e	dYBc	sBdBc - sBdBc
t9					-a	b									a	-b						-1		...	tBeAc - tBeBf
s9					c	d									-c	-d							s2e1c	d01	sBeAc - sBeBf
t10			-a	b	a	-b																-1		d02	tAcAb - tBfBa
s10			c	d	-c	-d																	s1c1b	...	sAcAb - sBfBa
																								dλ1	
																								dλ2	

6.4 PRECISION MEASURES AND CHECKS

6.4.1 General statistics

The program calculates and prints a variety of general statistics. These cover the number of points held fixed, the number of provisional points, the number of duplicate points referenced to either a fixed or a provisional point, and the number of trigonometrical reference beacons used. The total number of points is also printed. All of these statistics are determined by the program as it processes and counts the various data elements.

The standard deviation used for the distance observations in the EDM era is reported as 'Std Dev 1985 & Post', and the pre-EDM era standard deviation as 'Std Dev Pre 1985'.

The number of polygons, the number of survey records and the number of observations are also reported. This, together with information relating to the number of fixed and provisional points, is used to calculate the degrees of freedom in accordance with the formulae in Section 6.2.6.

6.4.2 Global check - direction observations

This has been done in the conventional manner. The contents of each field in the printout is as explained below opposite the appropriate printout column heading (see Appendix E.3):

From	The sequential number of the point from which the observation is made.
To	The sequential number of the point to which the observation is made.
Refer	The nature of the point, with F for fixed, P for provisional, and R for referenced together with the sequential number of the point referenced to. A referenced point has been identified as representing the same point as the point referenced to. These are common points between polygons.
Observation	The joined direction between the provisional points.
Res	The residual determined in the adjustment.
Orient Corr	The correction to the provisional orientation correction.

Corrected	The observation plus the residual plus the orientation correction.
Join	The joined direction between the adjusted coordinates.
Diff	The difference between the corrected observation and the join between the adjusted coordinates.
Weight	The weight assigned to the observation in the adjustment.
Residual (metres)	The residual converted to metres as a function of the base as explained in Section 6.2.5.

6.4.3 Global check - distance observations

This has been done in the conventional manner. The contents of each field in the printout are as explained below opposite the appropriate printout column heading:

From	The sequential number of the point from which the observation is made.
To	The sequential number of the point to which the observation is made.
Refer	The nature of the point, with F for fixed, P for provisional, and R for referenced together with the sequential number of the point referenced to. A referenced point has been identified as representing the same point as the point referenced to. These are common points between polygons.
Observation	The joined distance between the provisional points.
Res	The residual determined in the adjustment.
Scale factor	The correction to the provisional scale factor.
Corrected	The observation plus the residual multiplied by the scale factor.
Join	The joined distance between the adjusted coordinates.
Diff	The difference between the corrected observation and the join between the adjusted coordinates.
Weight	The weight assigned to the observation in the adjustment.

6.4.4 Standard deviations

The standard deviation of the observation of unit weight (σ_0) apriori (estimated before

the adjustment) and aposteriori (determined in the adjustment) are both printed out. The apriori estimate is that which was used in creating the weight model.

The aposteriori estimate gives an indication of how correct the initial estimate was, of the precision of the data as reflected by the standard deviation. A value smaller than the apriori estimate indicates that the observations were more precise than was assumed, while a larger value indicates that they were less precise.

6.4.5 Adjusted coordinates and error ellipses

An error ellipse is a statistical measure indicating the size, shape and orientation of an area, within which one would expect the solution to lie, with a certain level of confidence. The ellipse is specified at a certain confidence level by its semi-major ' a ' and semi-minor ' b ' axes, and the orientation of its semi-major axis ' θ '. Standard error ellipses at a 39.4% level of confidence have been used. Noting that $Q = (A'wA)^{-1}$, a , b and θ may be expressed as:

$$a^2 = \sigma_0^2/2 (Q_{xx} + Q_{yy} + ((Q_{xx} - Q_{yy})^2 + 4Q_{xy}^2)^{1/2}) \quad (14)$$

$$b^2 = \sigma_0^2/2 (Q_{xx} + Q_{yy} - ((Q_{xx} - Q_{yy})^2 + 4Q_{xy}^2)^{1/2}) \quad (15)$$

$$\tan 2\theta = 2Q_{xy} / (Q_{xx} - Q_{yy}) \quad (16)$$

The geometry of the network, together with the standard deviations of the observations, usually determines the size, shape and orientation of the ellipse. Conceptually the ellipse is smallest in the direction that is constrained by the highest level of certainty, or in other words, where the observations are more precise.

The influence of a direction observation in a polar constrains a solution perpendicular to the influence of a distance observation. If the distance observation were more precise than the direction observation then one would expect the semi-minor axis of the ellipse to be oriented in the direction of the observation, and the semi-major axis perpendicular to the observation.

If numerous observations were made to a particular point, then the geometry of the network is an important determinant of the orientation and dimension of the ellipses.

In the mathematical model employed in this study, the direction and distance observations are derived from the same set of coordinates, and as explained in Section 6.2.4, they must then be assigned equivalent weight in the adjustment. This means that they have equal and perpendicular influences on the error ellipse, which then becomes a circle. This is confirmed by the results because the semi-major and semi-minor axes, ' a ' and ' b ', were computed equal for a point in all instances. Where there was more than one observation to a point, and where there were influences of differing precision the ellipses computed differently in size from point to point, but always with ' a ' equal to ' b ' for a point. Geometrically these are then circles of differing radius. This is in itself a check on the results, for were a point to compute different ' a ' and ' b ' values then some error is likely, which would need to be detected.

The adjusted coordinates and associated information are printed out under the following column headings:

Number	The sequential number of the point in the list.
Name	The name of the point as it appears on the source diagram.
Provisional Y & X	The provisional estimates of the coordinates.
Corrections Y & X	The unknowns determined in the adjustment. Where a point was referenced, this is stated together with the sequential number of the point it is referenced to.
Adjusted Y & X	The provisional coordinate plus the correction.
Std Deviations	The standard deviation in Y and X as DY and DX, followed by the vector sum of the two as dP.
Error Ellipse	The standard error ellipse parameters are given with ' a ' and ' b ' representing the semi-major and semi-minor axes respectively. Deg and Min give the orientation of the semi major-axis in degrees and minutes.

6.4.6 Orientation and scale correction

The orientation and scale corrections have been determined in the normal manner and reported together with the appropriate observations. A better impression of the significance of each correction is obtained when its effect on the polygon is seen.

This is done by converting the angular orientation change in the polygon to an equivalent distance in metres by using the determined radius of influence of the polygon as a base. The information is printed under the heading 'Affect on Poly' next to the orientation correction in seconds. The radius of influence was calculated as the distance from the centroid to the furthest point of the polygon. (See Section 4.2).

Similarly the effect of the scale factor is printed under the heading 'Affect on Poly' immediately following the determined scale factor.

6.4.7 Geometrical consistency check

The final check on the results has been described as the 'Geometrical Consistency Check' on the program printouts.

This records the difference between the angles at the polygon apices prior to adjustment and after adjustment. An angular change is difficult to interpret because its real effect is a function of the length of the sides on either side of it. A 10" change in an angle impacts on a side of 10m by an amount of 0.00048m, while the same change impacts on a side of 1000m by 0.048m.

A short side may lie on one side and a long side on the other of a particular apex. One would be misled in thinking that the angular change is significant to the long side, because the short side was also used in its determination. The significance of the change in an angle is best measured by its influence on the opposite side of the triangle, as this is a function of the length of both sides adjacent to the apex and the particular angle.

The printout therefore records the distance between the points adjacent to the particular apex angle being checked. The data distance (the join distance between the provisionals in this case study) and the data distance multiplied by the scale are both recorded.

The distance between the adjusted coordinates is then recorded and the difference between the scale corrected data distance and the 'adjusted' distance (between the adjusted coordinates) is recorded in metres.

This is then converted to an angular measure via radians, and recorded as seconds. The approach may not be conventional, or entirely rigorous. The author is convinced that it allows for quicker and easier assessment of the preservation of the polygon geometry than merely recording the changes in the angles at the apices. This is the ultimate goal of printing the particular check. One recommended change to the programming is the inclusion of the conventional angle measures, which have not been included in the program used in this study.

6.4.8 Correlation between unknowns

If high levels of correlation exist between the unknowns in a least squares solution then the variables being determined are shown to be dependent and not independent. This in itself does not imply that the solution is invalid, but would suggest that further theoretical analysis and practical testing are required. (See Section 8.5). No postulations relating to correlation were made or tested in this thesis. The correlation has been calculated merely to see if this is a potential problem area requiring further research.

$$\text{Correlation} = \frac{Q_{ij}}{\sqrt{Q_{ii}} \sqrt{Q_{jj}}} \quad (17)$$

The correlation between the unknowns in the *ith* row and *jth* row of the solution vector is obtained by dividing the element at the *ith* row and *jth* column of the $(A'wA)^{-1}$ matrix by the square root of the elements of the *ith* row and *jth* column of the main diagonal. The formula (17) above is used for this calculation, in which Q means $(A'wA)^{-1}$.

6.5 CHAPTER SUMMARY

In the mathematical model in this thesis, data are derived from the sides, angles, directions or coordinates on a diagram, and are treated as observations between the points representing the apices of the polygons. These are not true observations and are referred to as pseudo observations.

The degrees of freedom for the network under the first mathematical model may be defined as *Degrees of freedom* = $2(f + c - m)$. In other words as long as there are more known (f) and common (c) points than polygons (m) a solution should be possible under the first mathematical model.

The degrees of freedom for the network under the second mathematical model are defined as *Degrees of freedom* = $2(f + c - r)$. In other words as long as there are more known (f) and common (c) points than survey records (r) a solution should be possible.

The program printouts have the conventional items for a network such as the provisional coordinates, the solution vector, the final adjusted coordinates, the standard deviations of the coordinates, and the error ellipse parameters.

In the mathematical model employed in this study, the direction and distance observations are derived from the same set of coordinates. As explained in Section 6.2.4, they must then be assigned equivalent weight in the adjustment. This means that they have equal and perpendicular influences on the error ellipse, which then becomes a circle.

A global check on both direction and distance observations is included. This checks that the observation plus its residual and the orientation correction (or times the scale factor for distances) equate to the join direction and distance between the adjusted coordinates.

Two further checks have been programmed. The first lists the effects of the scale and orientation corrections on the polygon by applying it to the radius of influence of the polygon.

The second, a geometrical consistency check, compares the pre- and post-adjustment polygon apex angles.

The correlation between the adjusted unknowns has been calculated to determine if this is an obvious problem area requiring further research. No postulations were made or tested in this study.

CHAPTER SEVEN
SUPPORT PROGRAMS

CONTENTS

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